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Chemistry of Chern-Simons Supergravity: reduction to a BPS kink, oxidation to M-theory and thermodynamical aspects

Luzi Bergamin,[a*] **Daniel Grumiller,**[b†] **Alfredo Iorio**[cd‡] **and Carlos Nuñez**[c§]

[a] *Institut für Theoretische Physik, Technische Universität Wien*
Wiedner Hauptstr. 8-10/136, A-1040 Vienna, Austria
[b] *Institut für Theoretische Physik, Universität Leipzig*
Augustusplatz 10-11, D-04109 Leipzig, Germany
[c] *Center for Theoretical Physics, Massachusetts Institute of Technology*
77, Massachusetts Avenue - Cambridge MA 02139 U.S.A.
[d] *Istituto Nazionale di Fisica Nucleare, Rome, Italy*
E-mail: bergamin@tph.tuwien.ac.at, grumiller@itp.uni-leipzig.de, iorio@lns.mit.edu, nunez@lns.mit.edu

Abstract: We construct a supersymmetric extension of the two dimensional Kaluza-Klein-reduced gravitational Chern-Simons term, and globally study its solutions, labelled by mass and U(1) charge c. The kink solution is BPS, and in an appropriate conformal frame all solutions asymptotically approach AdS. The thermodynamics of the Hawking effect yields interesting behavior for the specific heat and hints at a Hawking-Page-like transition at $T_{\text{critical}} \sim c^{3/2}$. We address implications for higher dimensions ("oxidation"), in particular $D = 3, 4$ and 11, and comment briefly on AdS/CFT aspects of the kink.

Keywords: Chern-Simons Theories, M-Theory, 2D Gravity, Supergravity Models.

Dedicated to the people of Iraq.

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Contents

1. Introduction

The 3-dimensional (3D) gravity theory governed by the sole Chern-Simons (CS) term

$$S_{\rm CS} = \frac{1}{4\pi^2} \int {\rm d}^3x \epsilon^{\mu\nu\lambda} \left(\frac{1}{2} \Gamma^\rho_{\mu\sigma} \partial_\nu \Gamma^\sigma_{\lambda\rho} + \frac{1}{3} \Gamma^\rho_{\mu\sigma} \Gamma^\sigma_{\nu\tau} \Gamma^\tau_{\lambda\rho} \right) , \tag{1.1}$$

where $\Gamma^\lambda_{\mu\nu} = \frac{1}{2} G^{\lambda\rho}(\partial_\mu G_{\nu\rho} + \partial_\nu G_{\mu\rho} - \partial_\rho G_{\mu\nu})$, and $G_{\mu\nu}$ is the 3D metric tensor,[1] possesses, among others, an interesting kink solution [1], whose global properties were extensively

[1] We will use: $A, B, C, \ldots = 0, 1, 2$ for 3D flat indices, $\mu, \nu, \lambda, \ldots = \hat{0}, \hat{1}, \hat{2}$ for the curved ones; while $a, b, c, \ldots = 0, 1$ are the 2D flat indices, and $m, n, l, \ldots = \hat{0}, \hat{1}$ are the curved ones. When not misleading we will also use upper case letters for the 3D quantities, like the metric $G_{\mu\nu}$, its determinant G, and lower case for the 2D ones, like the metric g_{mn}, its determinant g. But we do not always use this convention, as for instance, the Dreibein is e^A_μ and its inverse E^μ_A. More notation and conventions for spinors are explained in appendix A.

investigated [2]. One of the main goals of this paper is to explore under which circumstances such solutions are BPS states within a supersymmetric extension and to connect them with other well studied systems.

The role of the CS term in 3D Einstein-Hilbert (EH) gravity

$$S_{\rm EH} = \kappa \int {\rm d}^3x \sqrt{G} R\,, \tag{1.2}$$

where R is the 3D scalar curvature, was first studied by Deser, Jackiw and Templeton (DJT) [3]. The DJT action

$$S_{\rm DJT} = S_{\rm EH} + \frac{1}{\mu} S_{\rm CS} \tag{1.3}$$

admits a local excitation φ of mass μ, while none of the two terms separately, $S_{\rm EH}$ or $S_{\rm CS}$, supports any local excitation due to their entirely topological nature.[2] Thus one might consider the theory (1.1) as the limiting case ($\mu \to 0$) of the theory (1.3).

Solutions of (1.1) were found in [1] by reducing the 3D theory down to two dimensions by means of a Kaluza-Klein *Ansatz*[3]

$$G_{\mu\nu} = \begin{pmatrix} g_{mn} - \varphi a_m a_n & -\varphi a_m \\ -\varphi a_n & -\varphi \end{pmatrix}, \tag{1.4}$$

where g_{mn} is the $D = 2$ metric tensor, a_m is a $D = 2$ gauge vector, and φ is a scalar (essentially the conformal factor). It is assumed that the system has an isometry such that in the adapted coordinate system implied by (1.4) all quantities are independent of one of the coordinates, which will be denoted by r. By expressing the Christoffel connection $\Gamma^{\lambda}{}_{\mu\nu}$ in terms of the spin connection $\omega_\mu{}^{AB}$, the latter—as torsion vanishes—in terms of Vielbein e_μ^A, using the 3D property $\omega_{\mu}{}_{,AB} = \epsilon_{ABC}\, \omega_\mu{}^C$, and $\omega_\mu{}^A = e_\mu^B \omega_B{}^A$ we have

$$\omega_A{}^D = \frac{1}{2} \epsilon^{BCD} \left(e_{\mu A} \partial_B E_C^\mu + e_{\mu B} \partial_A E_C^\mu + e_{\mu C} \partial_B E_A^\mu \right), \tag{1.5}$$

where $\eta_{AB} e_\mu^A e_\nu^B = G_{\mu\nu}$, and $e_\mu^A E_B^\mu = \delta_B^A$. With the *Ansatz* (1.4) the Dreibein and spin connection read

$$e_\mu^A = \begin{pmatrix} e_m^a & 0 \\ \sqrt{\varphi} a_m & \sqrt{\varphi} \end{pmatrix}, \tag{1.6}$$

$$\omega_m^a = \frac{1}{2} e_m^a \sqrt{\varphi} f + \frac{1}{2} a_m \frac{1}{\sqrt{\varphi}} \epsilon^{ab} E_b^n \partial_n \varphi\,, \tag{1.7}$$

$$\omega_m^2 = -\omega_m - \frac{1}{2} \varphi f a_m\,, \tag{1.8}$$

$$\omega_2^a = \frac{1}{2} \frac{1}{\sqrt{\varphi}} \epsilon^{ab} E_b^m \partial_m \varphi\,, \tag{1.9}$$

[2] While the topological nature of the CS term is self-evident, it is more subtle to spot the topological nature of the EH action in $D = 3$. On this see [4, 5, 6].

[3] In the present case the scalar field φ appears slightly differently from [1], the difference being a conformal transformation of the *reduced* metric. We will address this point again below.

$$\omega_{\hat{2}}^{2} = -\frac{1}{2}\varphi f\,, \tag{1.10}$$

respectively, where the 2D spin connection $\omega_{m,\,ab} = \epsilon_{ab}\,\omega_m$ and the dual field strength f

$$f_{mn} = \partial_m a_n - \partial_n a_m = \sqrt{-g}\,\epsilon_{mn} f \tag{1.11}$$

have been introduced ($g = \det g_{mn}$, recall that $G = \det G_{\mu\nu} = -\varphi(g)$).

The 2D theory obtained is [1]

$$\mathrm{CS} = -\frac{1}{8\pi^2}\int \mathrm{d}^2x\sqrt{-g}\varphi f(r - \Box \ln\varphi + \varphi f^2)\,, \tag{1.12}$$

where $r = 2\epsilon^{mn}\partial_m\omega_n$ is the 2D scalar curvature.

We digress here on the degrees of freedom of this theory. Off-shell, in D dimensions, a metric $G_{\mu\nu}$ has $D(D+1)/2 - D = D(D-1)/2$ degrees of freedom, being a symmetric rank-two tensor and the action being invariant under D reparametrizations. This gives 3 degrees of freedom in $D = 3$ (and 1 in $D = 2$). The theory (1.1) also enjoys conformal invariance[4] as a consequence of the eqs. of motion which imply the vanishing of the Cotton tensor

$$2\sqrt{g}C^{\mu\nu} = \epsilon^{\mu\sigma\tau}D_\sigma R^\nu_\tau + \epsilon^{\nu\sigma\tau}D_\sigma R^\mu_\tau\,, \tag{1.13}$$

where D_μ is the covariant derivative based on ω_μ^{AB}. For this 3D action the true degrees of freedom off-shell are only 2 as we can get rid of the conformal factor φ by setting it to 1. This can be seen by performing a conformal transformation of the *reduced* theory, $g_{mn} \to \varphi g_{mn}$, assuming positive φ (as the quantity φ has to be nonvanishing for a non-degenerate metric (1.4) this is not an additional restriction). As the gauge field a_m remains unchanged, the field strength f behaves inversely to the volume form $f \to f/\varphi$. With the well-known relations

$$\sqrt{-g}\,r \to \sqrt{-g}\,r + \sqrt{-g}\,\Box\ln\varphi \tag{1.14}$$

and $\sqrt{-g} \to \varphi\sqrt{-g}$ one obtains from (1.12) the simpler action

$$\mathrm{CS} = -\frac{1}{8\pi^2}\int \mathrm{d}^2x\sqrt{-g}(fr + f^3)\,, \tag{1.15}$$

leaving 1 degree of freedom for r and 1 for a_m (recall that the performed reduction does not change the total number of degrees of freedom, but simply rearranges them). Obviously, the scalar field φ can play no role as the action (1.15) is *independent* thereof.[5] On the other hand, conformal invariance does not hold in the EH sector[6] as can be seen easily by reducing the action (1.2) with the *Ansatz* (1.4)

$$\mathrm{EH} \sim \int \mathrm{d}^2x\sqrt{-g}\sqrt{\varphi}\left(r + \varphi\frac{1}{2}f^2\right)\,, \tag{1.16}$$

hence the degrees of freedom off-shell are three: one for r, one for a_m, and one for φ.

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[4]That is why, sometimes, in literature the Chern-Simons gravitational term is also referred to as "conformal gravity" [7].

[5]We thank Roman Jackiw for helpful discussions on this point.

[6]Nonetheless, it is customary in standard Kaluza-Klein reduction of the EH term to set $\varphi = 1$ *in the action*. This is to avoid a meaningless theory by setting $\varphi = 1$ in the Euler-Lagrange eqs. On this point see, for instance, [8].

JHEP11(2004)021

As well known, this discussion is valid *off-shell*, as *on-shell* neither CS in (1.1) nor EH in (1.2), separately, have any degree of freedom. It is interesting to note that classically a conformal transformation on the 2D metric is always possible and this gives another perspective on the origin of the dynamical (on-shell) degree of freedom of DJT: by virtue of (1.14) the kinetic term for $\ln\varphi$ can never disappear when *both* terms (1.12) and (1.16) are considered. We might call this mode a "conformal mode".

In [1] solutions of (1.15) were found, with field strengths

$$f = 0 \quad (\mathrm{A})\,, \qquad f = \pm\sqrt{c} \quad (\mathrm{B})\,, \qquad f = \sqrt{c}\tanh\frac{\sqrt{c}}{2}x \quad (\mathrm{C})\,, \tag{1.17}$$

corresponding scalar curvatures

$$r = c \lesseqgtr 0 \quad (\mathrm{A})\,, \qquad r = -2c < 0 \quad (\mathrm{B})\,, \qquad r = -2c + \frac{3c}{\cosh^2\frac{\sqrt{c}}{2}x} \quad (\mathrm{C})\,, \tag{1.18}$$

and line elements

$$(\mathrm{d}s)^2 = \frac{2}{ct^2}[(\mathrm{d}t)^2 - (\mathrm{d}x)^2] \quad c > 0 \quad (\mathrm{A})\,, \tag{1.19}$$

$$(\mathrm{d}s)^2 = \frac{2}{|c|x^2}[(\mathrm{d}t)^2 - (\mathrm{d}x)^2] \quad c < 0 \quad (\mathrm{A})\,, \tag{1.20}$$

$$(\mathrm{d}s)^2 = \frac{1}{cx^2}[(\mathrm{d}t)^2 - (\mathrm{d}x)^2] \quad c > 0 \quad (\mathrm{B})\,, \tag{1.21}$$

$$(\mathrm{d}s)^2 = \frac{1}{\cosh^4\frac{\sqrt{c}}{2}x}(\mathrm{d}t)^2 - (\mathrm{d}x)^2 \quad c > 0 \quad (\mathrm{C})\,. \tag{1.22}$$

By using a convenient coordinate system the (kinky) spacetime in (C) may be extended beyond $x = \pm\infty$, as the affine distance to this "boundary" is finite (see [2] and eq. (3.21) below).

In the following sections we intend to construct a suitable supersymmetric theory compatible with (at least some) supersymmetry (SUSY) of the line elements (1.21) and (1.22). The paper is organized as follows: section 2 is devoted to the construction, via the rigorous methods of graded Poisson Sigma Models (gPSM), of the suitable SUSY for the 2D model (1.15). There we present our supersymmetric CS (SUCS) 2D lagrangian and transformations, and we show the differences with the theory obtained by reducing the 3D SUCS. In section 3 we study the SUSY of the solutions and discuss their global properties. In section 4 we compare our 2D model to the 3D model of [9] and we spell a connection with 4D supergravity and M theory. Finally section 5 contains our conclusions. Appendix A explains notation and conventions of the 2D formulation, appendix B provides details on the formulation of SUCS as a gPSM, and appendix C discusses conformal transformations of the reduced theory.

2. Construction of the 2D SUGRA model

To identify the conditions under which the metrics (1.21) and (1.22) preserve some SUSY we will follow the usual prescription of finding covariantly constant local SUSY parameters

$\epsilon_\alpha(x)$ (see for example [10, 11]) satisfying

$$\hat{D}_\mu \epsilon_\alpha = (D_\mu + F_\mu)\epsilon_\alpha = \partial_\mu \epsilon_\alpha + \omega_\mu^{AB} \left(\Gamma_{AB}\right)_\alpha^\beta \epsilon_\beta + (F_\mu \epsilon)_\alpha = 0\,, \tag{2.1}$$

where D_μ is the covariant derivative based on ω_μ^{AB}, Γ_{AB} are the local Lorentz generators, F_μ is a one form that will be determined below and is related to additional fields present in the system. We have not specified yet the dimension D of the spacetime. The general idea is that on a curved manifold it is not possible in principle to define such spinors, because when parallel transported, the spinor transforms as

$$\epsilon(x) = e^{i\int_0^x \omega} \epsilon(0)\,, \tag{2.2}$$

so, in a closed path $\epsilon(0) = e^{i\oint \omega}\epsilon(0)$, in general the spinor is not uniquely defined. But it might happen that other terms do contribute to the parallel transport, such that $\epsilon(0) = e^{i\oint(\omega+F)}\epsilon(0)$, being F the one form related to F_μ above. When a cancellation between ω and F occurs, the spinors are compatible with (some fraction of) SUSY, hence the given field configuration is (partially) supersymmetric.

We will be interested in purely bosonic configurations with vanishing fermionic variations, as bosonic SUSY variations will vanish trivially in these cases (generalizations of this concept are discussed briefly below). This is why it is usually stated that a bosonic field configuration is supersymmetric if the fermionic variations vanish. Examples of this are all the extremal Dp branes in type-II SUGRA theories and extremal M-branes in $D = 11$ SUGRA.

Another way of looking at eq. (2.1) is to consider the supergravity (SUGRA) transformations

$$\begin{aligned} \delta e_\mu{}^A &= -2i(\varepsilon\Gamma^A\psi_\mu)\,, & (2.3)\\ \delta\psi_{\mu\alpha} &= -\hat{D}_\mu\varepsilon_\alpha\,, & (2.4)\end{aligned}$$

in the case when the Rarita-Schwinger fermions $\psi_{\mu\alpha}$ (partners of the bosonic Vielbein $e_\mu{}^A$) and their variations vanish [11]. In this case eq. (2.1) is a consequence of eq. (2.4). For the 3D DJT model a SUGRA extension is available [9], and the SUGRA multiplet is as given in (2.3)–(2.4).

2.1 The SUGRA multiplet

We will now exploit these methods and propose the following SUGRA multiplet and transformations for the 2D theory (1.15)

$$\begin{aligned} \delta e_m{}^a &= -2i(\varepsilon\gamma^a\psi_m)\,, & (2.5)\\ \delta a_m &= -2\varepsilon\gamma_*\psi_m\,, & (2.6)\\ \delta\psi_{m\alpha} &= -\hat{D}_m\varepsilon_\alpha & (2.7)\end{aligned}$$

where

$$\hat{D}_m\varepsilon_\alpha = \partial_m\varepsilon_\alpha + \frac{1}{2}\tilde{\omega}_m(\gamma_*\varepsilon)_\alpha + \frac{i}{2}F(\gamma_m\varepsilon)_\alpha\,, \tag{2.8}$$

JHEP11(2004)021

$$F = f + \epsilon^{mn}\psi_n\gamma_*\psi_m\,. \tag{2.9}$$

In $D=2$ $\frac{1}{2}\gamma_*$ is the generator of Lorentz transformations in spinor space, $\gamma_m = e_m^a\gamma_a$, and the torsionfull connection reads ($\tilde{\omega}_m = e_m^a\tilde{\omega}_a$, more details on the notation are explained in appendix A)

$$\tilde{\omega}_a = \epsilon^{mn}\partial_n e_{ma} - i\epsilon^{mn}(\psi_n\gamma_a\psi_m)\,. \tag{2.10}$$

Immediately one sees that the balance of fermionic and bosonic degrees of freedom is correct: the 2 bosonic degrees of freedom (1 for e_m^a and 1 for a_m) are transformed into 2 fermionic degrees of freedom ($\psi_{m\alpha}$ has 4 independent components to which we must subtract 2 components as consequence of the irreducibility condition $\gamma_m\psi^m = 0$, leaving 2 fermionic degrees of freedom).

The transformation of the Zweibein is customary, as well as the transformation of the Rarita-Schwinger field (when the previous argument on the "improved" covariant derivative is assumed), although the exact expression for the extra term $\frac{i}{2}F(\gamma_m\varepsilon)_\alpha$ will be fully justified below in the gPSM construction. What is more subtle is the transformation law for the gauge field, that might look rather unusual. Our qualitative argument for it is based on the conformal symmetry of the Chern-Simons term (1.1): from the Kaluza-Klein split of the Dreibein (1.6) and the SUGRA transformations (2.3) one sees that the components $e_m^2 = \sqrt{\varphi}a_m$ transform into something proportional to $\varepsilon\gamma_*\psi_m$. This would be true also for the EH sector of the DJT theory, but for the CS sector we can invoke conformal symmetry and set $\varphi = 1$.

To prove that our *Ansatz* (2.5)–(2.9) is viable we must now supersymmetrize (1.15). This could be done in different ways, for instance using the metric as the basic geometric variable (second order formalism). We will, instead, employ as basic objects the Zweibein and connection (first order formalism), as for dilaton (super-)gravity in $D=2$ this approach provides powerful tools to discuss classical and quantum aspects of the theory (for a review cf. [12] and refs. therein). Once this formalism is incorporated classical solutions may be obtained *globally* with particular ease. These successes are intimately related to the fact that it is essentially a special case of a (graded) Poisson Sigma Model (gPSM) [13], that we will now briefly introduce (for a more comprehensive review cf. e.g. [14]).

2.2 gPSM supersymmetrization: the action

The action of a gPSM reads[7]

$$\mathcal{S}_{gPSM} = \int_{\mathcal{M}_2} \mathrm{d}X^I\wedge A_I + \frac{1}{2}P^{IJ}A_J\wedge A_I\,. \tag{2.11}$$

[7] Geometrically, there is a 2D base manifold $\mathcal{M}_2$ and a target space $\mathcal{N}$, the latter being a Poisson manifold with associated Poisson tensor P^{IJ} and with coordinates denoted by X^I (indices $I,J,K,\dots$ run from 1 to the dimension of the target space). Those coordinates as well as the gauge fields A_I are functions of the coordinates x^m on the base manifold, $X^I(x)$ and $A_I(x)$. The same symbols are used to denote the mapping of $\mathcal{M}_2$ to $\mathcal{N}$. The $\mathrm{d}X^I$ stand for the pullback of the target space differential $\mathrm{d}X^I = \mathrm{d}x^m\partial_m X^I$ and A_I are 1-forms on $\mathcal{M}_2$ with values in the cotangent space of $\mathcal{N}$. It is sometimes convenient to interpret the gauge field as a 1-form not only with respect to the base manifold but also with respect to the target space, $\mathcal{A} = \mathrm{d}X^I\wedge A_I$. It has been shown that a gPSM arises as the most general consistent deformation (*à la* Barnich and Henneaux [15]) of 2D BF theory [16].

The name "Poisson Sigma Model" is justified because it is a Sigma model (the target space coordinates behave as scalar fields from the world-sheet point of view) and because the target space is a Poisson manifold. "Graded" refers to the fact that one allows for a $\mathbb{Z}_2$ grading of the target space in order to accommodate SUSY. Thus, a gPSM is defined by the specification of the Poisson tensor P^{IJ} depending on the target space coordinates X^K: its dimension, its maximal rank, its $\mathbb{Z}_2$ grading properties and its entries. For theories describing gravity generically it has a nontrivial kernel and thus it is not symplectic.

As a consequence of the graded non-linear Jacobi identities

$$P^{IL}\partial_L P^{JK} + \textit{g-perm}\,(IJK) = 0\,, \tag{2.12}$$

where *g-perm* stands for all graded cyclic permutations, a gPSM (2.11) is invariant under the nonlinear symmetry transformations with symmetry parameters $\varepsilon_I(X^J, x^m)$

$$\delta X^I = P^{IJ}\varepsilon_J\ , \tag{2.13}$$

$$\delta A_I = -\mathrm{d}\varepsilon_I - \left(\partial_I P^{JK}\right)\varepsilon_K\, A_J\,, \tag{2.14}$$

where $\partial_I := \partial/\partial X^I$. The symmetries (2.13), (2.14) comprise local Lorentz transformations, diffeomorphisms and local SUSYs, respectively.

The identities (2.12) pose non-trivial constraints on the possible form of the Poisson tensor P^{IJ}. If P^{IJ} is linear the otherwise non-linear gauge transformations (2.14) reduce to ordinary non-abelian ones; the term $\partial_I P^{JK}$ yields the structure constants which also enter the eqs. of motion. They are obtained by varying (2.11) with respect to X^I and A_I, and are of first order. A simple (hamiltonian) analysis shows that the number of physical propagating field degrees of freedom is zero. In this sense, the theory is a topological field theory of Schwarz type, cf. [17] for a review.

The absence of propagating physical modes does not necessarily imply that the model is trivial. In the case of gravity the non-trivial features are encoded in the number and types of (Killing) horizons and singularities as well as in the asymptotic behavior and the properties of geodesics of test particles (locally the model indeed is trivial as every 2D metric is conformally flat in a certain patch).

To derive the gPSM of interest in the present case we can rely on the knowledge of such models describing $N = (1, 1)$ dilaton SUGRA.[8] They require three bosonic (the "dilaton" X which can be interpreted as Lagrange multiplier for curvature and the Lagrange multipliers for torsion X^a) and two fermionic (the "dilatini" χ^α) target space coordinates. It is emphasized that this target space is quite different from the "standard target space" of string theory. In particular, as mentioned before, one of the target space coordinates corresponds to what in the string literature is referred to as "dilaton field" (cf. footnote 9).

[8] It should be emphasized, as discussed in refs. [18, 19], that not every gPSM which in its bosonic limit reduces to dilaton gravity [20] can be considered as genuine dilaton SUGRA [21]. Thus, dilaton SUGRA is not merely a gPSM, but a gPSM with a little bit of extra structure — this should not come as a surprise because also in the bosonic theory one needs extra structure for a PSM to be a (first order) dilaton gravity model.

JHEP11(2004)021

There are only two free functions of the dilaton field X defining the model.[9] Appealing to conformal transformations eliminates one of these and thus, neglecting subtleties with singular conformal transformations and quantum inequivalence, one has to specify only the prepotential $u(X)$ to be defined below. Thus, the art is to find the correct prepotential. Once this is achieved standard methods can be applied to solve the model locally and globally.

To each target space coordinate X^I corresponds a gauge field A_I: The bosonic spin-connection ω and Zweibeine e_a; the fermionic gravitini ψ_α. To describe the bosonic sector of SUCS [1] we need an extra bosonic target space coordinate [2] which we denote here by Y. The corresponding abelian gauge field 1-form is denoted by A. The prepotential may thus depend not only on X but also on Y.

It is emphasized again that once the prepotential is chosen everything is fixed and all classical solutions may be obtained globally. Without further ado we present the correct prepotential

$$u = X^2 - Y\,, \tag{2.15}$$

and explain in appendix B why it is the correct one and how it enters the Poisson tensor. Exploiting the relation between prepotential and potential [20],

$$V = -\frac{1}{8}\frac{\mathrm{d}}{\mathrm{d}X}u^2 = \frac{1}{2}(XY - X^3)\,, \tag{2.16}$$

eq. (2.16) is seen to be equivalent to eq. (3) in [2] without any obstruction.[10] This is necessary for a meaningful description of SUCS but not sufficient yet; the missing information is contained in the gauge transformations discussed in appendix B which is recommended for the reader interested in the relevant steps performed in the first order formulation. As crucial result A does not acquire a SUSY partner and does not transform as a gauge field (cf. eq. (B.23)), but rather as a component of the Dreibein, as expected from the theory in $D = 3$. This is quite non-trivial as we have adjusted the prepotential solely to establish the correct bosonic potential, but nonetheless the "miracle" happens and it provides also the transformation law for A we proposed in (2.6).

The main results of appendix B are the first order formulation of SUCS — which then allows to apply the discussion of [22] to study BPS solutions — and the transformation to the classically equivalent second order formulation

$$\mathcal{S}^I_{\text{SUCS}} = -\frac{1}{8\pi^2}\int \mathrm{d}^2x\sqrt{-g}\Big(\tilde{r}F + F^3 + \Sigma^2 - F^2\Delta\Big) \tag{2.17}$$

[9]For the reader familiar with scalar tensor theories it may be helpful to note that these two free functions of the target space coordinate X are *equivalent* to the ones appearing in the bosonic second order action

$$\int_{\mathcal{M}_2} \mathrm{d}^2x\sqrt{-g}\left(XR - Z(X)(\nabla X)^2 + 2V(X)\right).$$

Note also that in non-gPSM literature often an exponential representation of the dilaton field is used, $X = e^{-2\Phi}$.

[10]This is not always the case as obviously not every potential $V(X,Y)$ may be expressed in terms of a prepotential $u(X,Y)$.

JHEP11(2004)021

with $\tilde{r} = 2\epsilon^{mn}\partial_m \tilde{\omega_n}$, $\Sigma_\alpha = 2\epsilon^{nm}\hat{D}_m \psi_{n\alpha}$, $\Delta = \epsilon^{mn}\psi_n\gamma_*\psi_m$, and with the SUSY transformations (2.5)–(2.7).

It is interesting to note that the action (2.17) can be obtained from superspace techniques as well.[11] As our further calculations do not rely on superspace formulations we simply present the final result at this place and outline the derivation in appendix B. Two-dimensional supergravity in superspace [23] is formulated in terms of a scalar superfield S with field content $(e_m^a, \psi_m^\alpha, \mathcal{A})$. $\mathcal{A}$ is the auxiliary field and appears as lowest component in S. The action (2.17) is equivalent to the superspace action (E denotes the superdeterminant of the super-Vielbein)

$$\mathcal{S}_{\text{SP}} = \int \mathrm{d}^2x\, \mathrm{d}^2\theta E S^2 \tag{2.18}$$

if the auxiliary field is identified with the $\mathcal{A} = -F$.

2.3 A different supersymmetrization

Although the following sections will be devoted to the study of the model just constructed, we want to mention now a different (if not more direct) strategy that is to reduce the 3D SUCS action and transformations available in literature [9]. We will present here the results of such a reduction, show that it is not equivalent to the model just constructed, and leave its study to a future work.

From the Kaluza-Klein *Ansatz* (1.4) with $\varphi = 1$ and the SUSY variation (2.3) it follows that one has to choose $\psi_{\hat{2}\alpha} = 0$ in accordance with the discussion above. Then the reduced supersymmetric spin-connection is equivalent to the bosonic result (1.7)-(1.10) if the 2D spin-connection ω_m and f are replaced by their SUSY covariant versions $\tilde{\omega}_m$ in (2.10) and F in (2.9), respectively. The SUSY transformations of the reduced bosonic variables ${e_m}^a$ and a_m coincide with (2.5) and (2.6), while the transformations of the spinors become

$$\begin{aligned} \delta\psi_{m\alpha} &= -\hat{D}_m\varepsilon_\alpha + \frac{1}{4}F\big(i(\gamma_m\varepsilon)_\alpha - a_m(\gamma_*\varepsilon)_\alpha\big)\,, && (2.19)\\ \delta\psi_{\hat{2}\alpha} &= \frac{1}{4}F(\gamma_*\varepsilon)_\alpha\,. && (2.20)\end{aligned}$$

In contrast to the supersymmertization discussed above, the conformal frame defined by ${e_{\hat{2}}}^2 = 1$, $\psi_{\hat{2}} = 0$ is not supersymmetric, as can be seen from (2.20). Thus a suitable Wess-Zumino gauge, where the SUSY transformations are combined with an appropriate conformal transformation, should be implemented.

With this in hand, the reduction of the 3D SUCS action is straightforward. In the notation of (2.17) the result can be written as

$$\begin{aligned} \mathcal{S}^{II}_{\text{SUCS}} &= -\frac{1}{8\pi^2}\int \mathrm{d}^2x\sqrt{-g}\Big(\tilde{r}F + F^3 + \Sigma^2 + 2iF(\Sigma\gamma^m\gamma_*\psi_m) - \frac{1}{2}F^2\psi^m\psi_m\Big) && (2.21)\\ &= -\frac{1}{8\pi^2}\int \mathrm{d}^2x\sqrt{-g}\Big(\tilde{r}F + F^3 + \Sigma^2 - \frac{1}{2}F^2\Delta\Big)\,, && (2.22)\end{aligned}$$

[11] We thank the referee for drawing our attention to this point.

where (2.22) is obtained from (2.21) once the irreducibility constraint for Rarita-Schwinger fields, $\psi^a\gamma_a = 0$, is implemented.[12]

The theory (2.22) with transformations (2.5), (2.6) and (2.19) is not equivalent to the theory (2.17) with transformations (2.5)–(2.7), as the fermionic transformations and potential differ.

3. SUSY of the solutions

We now have all the ingredients to see under which conditions the solutions of the 2D theory are supersymmetric. According to eq. (2.7) we are looking for solutions of

$$\delta\psi_{m\alpha} = -\partial_m\epsilon_\alpha + \frac{1}{2}\omega_m\,(\gamma_*)_\alpha^{\ \beta}\,\epsilon_\beta - i\frac{f}{2}e_m^{\ a}\,(\gamma_a)_\alpha^{\ \beta}\,\epsilon_\beta = 0\,, \tag{3.1}$$

where only the bosonic contributions f and ω_m (to F and $\tilde{\omega}_m$, resp.) are important. We find that: no SUSY exists for the line elements (1.19) and (1.20), while (1.21) and (1.22) support half SUSY. We outline these results in some details, starting from the important case of the kink (1.22).

In a more general setting the line element (1.22) is written in the form (note our signature $(+,-)$)

$$(\mathrm{d}s)^2 = A(x)^2(\mathrm{d}t)^2 - (\mathrm{d}x)^2\,. \tag{3.2}$$

The only nonzero component of the spin-connection for this metric is $\omega_{\hat{0}} = \mathrm{d}A(x)/\,\mathrm{d}x \equiv A'$, hence the $m = \hat{1}$ component of eq. (3.1) is

$$\partial_x\epsilon + \frac{i}{2}f(x)\gamma_1\epsilon = 0\,, \tag{3.3}$$

and the $m = \hat{0}$ component is

$$\partial_t\epsilon - \frac{A'(x)}{2}\gamma_*\epsilon + \frac{i}{2}A(x)f(x)\gamma_0\epsilon = 0\,, \tag{3.4}$$

where the γ matrices are representation independent and written in flat indices (note that $\gamma_0\gamma_1 = \gamma_*$). We can assume that the spinor (like the full configuration) is time independent. We also need to impose one projection on the spinor, this preserves just half SUSY

$$\gamma_1\epsilon = -i\rho\epsilon\,, \qquad \rho = \pm 1\ . \tag{3.5}$$

Then the solutions of (3.4) are

$$\epsilon(x) = e^{-\rho/2\int_0^x \mathrm{d}x' f(x')}\epsilon(0) \tag{3.6}$$

with,

$$\frac{\rho A'}{A} = f \tag{3.7}$$

[12] This constraint is not supersymmetric for the model at hand and a WZ gauge should be used. However, it is used for illustrational purposes only and is never implemented in the remainder of this paper.

JHEP11(2004)021

and $\epsilon(0)$ being a constant spinor satisfying the projection (3.5). We see that the kink falls in this class, with $A(x)$ and $f(x)$ given in (1.22) and in (1.17)-(C), respectively, because it satisfies the constraint (3.7) with $\rho = -1$, explicitly

$$\epsilon(x) = \frac{\epsilon(0)}{\cosh \frac{\sqrt{c}}{2} x} \,. \tag{3.8}$$

According to (2.2) and the discussion below this can also be written as

$$\epsilon(x) = e^{i \int\limits_0^x (\omega - F)} \epsilon(0) \,, \tag{3.9}$$

where F is the matter contribution to the parallel transport given by the third term in eq. (3.1).

The symmetry breaking solutions $f = \pm\sqrt{c}$ ((1.17) (B) and line element (1.21)) also admit half SUSY. The last two cases, the line elements (1.19) and (1.20), show no SUSY because one of the two eqs. (3.1) requires ϵ_α *independent* from t (from x) for $c > 0$ (for $c < 0$), while the other equation requires that ϵ_α has to *depend* on t (on x), leading to a contradiction.

As the solutions (1.19)–(1.22) were found in [1] by a local analysis and by tuning one of the two constants of motion to a definite value, the considerations above are valid only in this sense. However, the solutions may be extended globally [2], and we will show this in the next subsection in particular for those preserving half SUSY.

3.1 All classical solutions

3.1.1 Bosonic part

For the convenience of the reader not so familiar with dilaton gravity [12] we recall briefly how to obtain bosonic solutions locally in the first order formalism and how to extend them globally by means of geodesic extension (for more details and further refs. cf. [2]). To this end one has to solve the eqs. of motion (B.9), (B.10), which read explicitly (dropping all wedges)

$$\mathrm{d}X - X^b \epsilon_b{}^a e_a + \frac{1}{2}(\chi\gamma_*\psi) = 0 \,, \tag{3.10}$$

$$DX^a + \frac{1}{2}\epsilon^{ab} e_b \big(XY - X^3 + \frac{1}{4}\chi^2\big) + \frac{i}{2} X(\chi\gamma^a\psi) = -W^a \,, \tag{3.11}$$

$$D\chi^\alpha - \frac{i}{2}X(\chi\gamma^a)^\alpha e_a + 2iX^a(\psi\gamma_a)^\alpha - (X^2 - Y)(\psi\gamma_*)^\alpha = 0 \,, \tag{3.12}$$

$$\mathrm{d}Y = 0 \,, \tag{3.13}$$

$$\mathrm{d}\omega + \frac{1}{2}\epsilon\big(Y - 3X^2\big) - \frac{i}{2}(\chi\gamma^a e_a\psi) - X(\psi\gamma_*\psi) = -W \,, \tag{3.14}$$

$$De_a + i(\psi\gamma_a\psi) = 0 \,, \tag{3.15}$$

$$D\psi_\alpha + \frac{1}{4}\epsilon\chi_\alpha - \frac{i}{2}X(\gamma^a e_a\psi)_\alpha = -W_\alpha \,, \tag{3.16}$$

$$\mathrm{d}A + \frac{1}{2}\epsilon X + \frac{1}{2}\psi\gamma_*\psi = 0 \,. \tag{3.17}$$

We recall the definition of the 0-form fields: X, X^a, χ^α, Y are dilaton, Lagrange multipliers for torsion, dilatino and U(1) charge, respectively, and the gauge field 1-forms: $\omega, e_a, \psi_\alpha, A$ are spin connection, dyad (related to the Zweibein), gravitino and U(1) connection, respectively. In the absence of matter the energy-momentum 1-forms $W^a = W_\alpha = 0$ vanish identically; the 2-form W is zero as well in that case, but it vanishes already for minimal coupling, i.e., if the matter action does not depend on the dilaton X. For a given matter action $S_{(m)}$ in light-cone coordinates these quantities read $W^{\pm\pm} = \delta S_{(m)}/\delta e^{\mp\mp}$, $W = \delta S_{(m)}/\delta X$ and $W_\pm = \delta S_{(m)}/\delta \psi^\mp$. We add some comments regarding the physical meaning of the eqs. of motion: (3.10) allows to eliminate the auxiliary fields X^a as directional derivatives of the dilaton X; (3.11) contains the relevant information about the Casimir function related to mass (see (3.18) below) and corresponds to the "Einstein equations" in the sense that minimally coupled matter may enter these eqs. as a source term W^a; (3.12) is the fermionic counterpart of these "Einstein equations"; (3.13) exhibits charge conservation of the U(1) gauge field A and yields the second Casimir (see (3.19) below); (3.14) yields curvature (matter coupled non-minimally to the dilaton may enter with a source term W); (3.15) is the torsion condition; (3.16) provides the equation for gravitino propagation; (3.17) allows to express the dilaton on-shell as the dual field strength related to A plus a soul contribution.

The Poisson tensor has at least two commuting Casimir functions that can be chosen as

$$C = \frac{1}{2} X^a X_a - \frac{1}{8}(X^2 - Y)^2 + \frac{1}{8} X \chi^2 \,, \tag{3.18}$$

being related to the mass, and the charge

$$c = Y \,. \tag{3.19}$$

We fix an eventual additive constant in (3.18) by the requirement that $C = 0$ on states respecting both supersymmetries. This happens to be the case in the definition (3.18). Also, the Casimir C as defined by (3.18) and $\mathcal{C}^{(g)}$ as defined in [2] differ by $Y^2/8$ from each other. Finally, note that $C < 0$ for a positive mass.

Of course, in order to solve the bosonic part we set $\chi^\alpha = 0 = \psi_\alpha$, thereby simplifying (3.10)–(3.19) considerably. Indeed, as we will see in the next paragraph it becomes almost a triviality to construct the classical solutions extending over basic Eddington-Finkelstein patches.

Basic patches Let us start with an assumption: $X^{++} \neq 0$ for a given patch (the indices $++$ and $--$ refer to light cone components introduced in appendix A.).[13] If it vanishes a (Killing) horizon is encountered and one can repeat the calculation below with indices $++$ and $--$ swapped everywhere. If both vanish in an open region by virtue of (3.10) a

[13]To get some physical intuition as to what this condition could mean: the quantities X^a, which are the Lagrange multipliers for torsion, can be expressed as directional derivatives of the dilaton field by virtue of (3.10) (e.g. in the second order formulation a term of the form $X^a X_a$ corresponds to $(\nabla X)^2$). For those who are familiar with the Newman-Penrose formalism: for spherically reduced gravity the quantities X^a correspond to the expansion spin coefficients ρ and ρ' (both are real).

JHEP11(2004)021

constant dilaton vacuum emerges, which will be addressed separately below. If both vanish on isolated points the Killing horizon bifurcates there and a more elaborate discussion is needed [24]. The patch implied by this assumption is a "basic Eddington Finkelstein patch", i.e., a patch with a conformal diagram which, roughly speaking, extends over half of the bifurcate Killing horizon and exhibits a coordinate singularity on the other half.

In such a patch one may redefine $e^{++} = X^{++}Z$ with a new 1-form Z. Then (3.10) implies $e^{--} = \mathrm{d}X/X^{++} + X^{--}Z$ and the volume form reads $\epsilon = e^{++} \wedge e^{--} = Z \wedge \mathrm{d}X$. The $++$ component of (3.11) yields for the connection $\omega = -\mathrm{d}X^{++}/X^{++} + ZV(X,Y)$. One of the torsion conditions (3.15) then establishes $\mathrm{d}Z = 0$, i.e., Z is closed. Locally, it is also exact: $Z = \mathrm{d}u$. It is emphasized that, besides the two Casimir functions, this is the only integration needed! Thus, after these elementary steps one obtains already the line element

$$\mathrm{d}s^2 = 2e^{++}e^{--} = 2\,\mathrm{d}u\;\mathrm{d}X + 2X^{++}X^{--}\,\mathrm{d}u^2\,, \tag{3.20}$$

which nicely demonstrates the power of the first order gravity/gPSM formalism.

It should be recalled that the set of classical solutions is labelled by the two Casimir functions C in (3.18), corresponding to minus the mass, and c in (3.19), corresponding to the U(1) charge. Exploiting their explicit form the bosonic part[14] of the line element (in Eddington-Finkelstein gauge) may be presented as

$$\mathrm{d}s^2 = 2\mathrm{d}u\,\mathrm{d}X + K(X;C,c)\mathrm{d}u^2\,, \tag{3.21}$$

$$K(X;C,c) = 2C + \frac{(X^2-c)^2}{4}\,. \tag{3.22}$$

Evidently there is always a Killing vector $k\cdot\partial = \partial/\partial u$ with associated Killing norm $k \cdot k = K(X;C,c)$. Now the geometrical meaning of the quantity $X^{++}X^{--}$ is clear: if and only if this product vanishes a (Killing) horizon is encountered. Thus, the condition $X^{++} \neq 0$ implies that the solution (3.21), (3.22) is valid in a basic outgoing (ingoing) Eddington-Finkelstein patch, i.e., it extends over outgoing (ingoing) horizons, only, as opposed to (3.2) which does not extend over any kind of horizon. Global solutions can be obtained by gluing together these basic patches appropriately. Note that with the simple coordinate transformation $\mathrm{d}u = (2/c)(\mathrm{d}t - \mathrm{d}x/A)$, $\mathrm{d}X = (c/2)\,\mathrm{d}x$ and the identification $A = (2/c)\sqrt{K}$ the line element (3.21) is equivalent to the one in (3.2). The coordinate singularities in that transformation at $A = 0 = K$ should be observed. The (bosonic) scalar curvature becomes[15]

$$r = -\frac{\mathrm{d}^2K}{\mathrm{d}X^2} = c - 3X^2\,, \tag{3.23}$$

in accordance with (1.18) (recall that on-shell $X = f$). Obviously, solutions with constant curvature are only possible for the constant dilaton vacua discussed below. Note that r is independent of the "mass" C. This somewhat counter intuitive feature is a consequence

[14] The fermionic part of the solutions can be discussed along the lines of refs. [20, 19].

[15] While (3.21) with (3.22) is not valid for constant dilaton vacua, (3.23) extends to these special solutions.

of the chosen conformal frame, in which generic solutions locally yield the same bosonic geometry as the ground state.[16]

It is emphasized that (3.22) comprises *all* non-constant dilaton solutions, in particular the ones given above in (1.22) emerging as the special case $C = 0$ and describing the patch $X \in (-\sqrt{c}, \sqrt{c})$ (the explicit form of the coordinate transformation from (3.22) with $C = 0$ to (1.22) can be found in section 2.2 of [2]).

Global properties In order to obtain the global Carter-Penrose diagrams one has to investigate the behavior of geodesics at the various boundaries. The basic idea is that if a, say, null geodesic reaches with finite affine parameter a boundary which does not exhibit a curvature singularity then spacetime may be extended. It is not the intention of the present work to repeat this discussion; rather, the reader is referred to [2].

3.1.2 Switching on SUSY

The classification of SUSY solutions is based upon the discussion in ref. [22]. It exhausts all smooth solutions preserving (at least) half of the SUSYs. A relevant observation is that the solutions that preserve some SUSY are equivalent to those with a vanishing body of the Casimir function C. This result can be anticipated on general grounds as the body of C is proportional to the determinant of the fermionic part of the Poisson tensor (B.5). An interesting non-smooth solution, the BPS-kink, will be discussed as well.

Constant dilaton vacua There is a very special class of solutions[17] called "constant dilaton vacua", i.e., X = const. globally. For the current model these are the two $\mathbb{Z}_2$ symmetry breaking solutions (1.17) (B) and the symmetry preserving one (1.17) (A), all of them yielding constant curvature (in $D = 2$ and also in $D = 3$). Provided the fermions are set to zero the $\mathbb{Z}_2$ symmetry breaking solutions indeed preserve half of the SUSYs because they are ground state geometries (in the sense that $C = 0$), in accordance with our previous discussion below eq. (3.9). Curvature and dual field strength are given by (cf. case (B) in (1.17), (1.18))

$$r = -2c\,, \qquad f = \pm\sqrt{c}\,. \tag{3.24}$$

[16]This feature is well-known from generic dilaton gravity [12] if a special conformal frame is used. For instance, the 2D part of the Schwarzschild BH

$$(\mathrm{d}s)^2 = 2\mathrm{d}u\mathrm{d}r + \left(1 - \frac{2M}{r}\right)\mathrm{d}u^2\,,$$

by a conformal transformation can be mapped onto

$$(\mathrm{d}s)^2 = 2\mathrm{d}u\mathrm{d}\tilde{r} + \left(\frac{1}{2}\sqrt{\tilde{r}} - 2M\right)\mathrm{d}u^2\,,$$

which displays the same property as (3.22) regarding the mass. Reverting this argument it is possible to transform (3.21) to a conformal frame in which C is a factor in front of a curvature singularity, see appendix C.

[17]From the gPSM point of view these solutions are remarkable as they allow not only for two but for four bosonic Casimir functions, i.e., the bosonic part of the Poisson tensor has minimal rank.

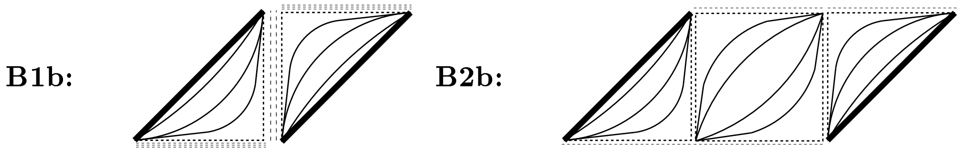

Figure 1: Basic Carter-Penrose patches for BPS solutions with non-constant dilaton.

The $\mathbb{Z}_2$ symmetry preserving one violates both SUSYs ($r = +c$, $f = 0$). The fact that symmetry breaking solutions preserve part of SUSY while symmetry preserving ones break it may have been expected on general grounds.

BPS states with non-constant dilaton In general, there exist two classes of BPS states:

Non-vanishing fermion fields. If the fermionic background is non-trivial for a BPS state the bosonic one has to be trivial, viz., Minkowski space [22]. However, none of the solutions (3.21) describe Minkowski space. Thus, there can be no BPS states with non-vanishing fermions.

Vanishing fermion fields. In this case only *the variation of the fermions* is important, as any variation of a boson is proportional to some fermion field that vanishes by assumption. All solutions with $C = 0$ are found to be BPS states provided the symmetry parameters are related in a specific way. The Killing norm is given by the complete square

$$K_{BPS}(X;c) = \frac{(X^2-c)^2}{4} = \frac{u^2}{4} \geq 0\,. \tag{3.25}$$

The expected [25] positivity result is recovered implying that all horizons have to be extremal. Curvature[18] and dual field strength read

$$r = c - 3X^2\,, \qquad f = X\,, \tag{3.26}$$

depending on the non-Killing coordinate $X \in (-\infty, \infty)$. The global structure has been discussed in [2]; keeping that notation[19] we recall the two Carter-Penrose diagrams with $C = 0$. Bold lines denote boundaries (in the present case they are curvature singularities), dashed lines Killing horizons (all of them are extremal here) and ordinary curved lines are hypersurfaces of constant f (see figure 1).

The former **B1b** arises as a special case of the latter **B2b** when the "kink"-region (the square patch in the middle) is shrunk to zero size by requiring $c = 0$. Note that both diagrams have to be rotated counter clockwise by 45^o if "time" is plotted vertically and "space" horizontally.[20] They can be extended to an infinite strip, much like the Carter-Penrose diagram of the Reissner-Nordström black hole (RN BH). Of course, one can identify periodically to get a finite strip; Möbius-strip like identifications are possible as well.

[18] Note that the curvature singularities are null complete but incomplete with respect to non-null geodesics. Thus, you cannot send an SMS with your mobile phone to it, but you can write it on a piece of paper and throw it there. This somewhat counter intuitive feature is a consequence of the chosen conformal frame.

[19] Cf. however the discussion below eq. (3.19). In figure 2 of that work BPS restricts to the curve starting from the origin of that diagram.

[20] In ref. [2] in section 3.1 in the third paragraph the words "time" and "space" have to be exchanged. Alternatively, of course one could rotate all Carter-Penrose diagrams in that work by 90^o.

JHEP11(2004)021

3.2 The BPS-kink

There is an interesting non-smooth candidate for a BPS state: the kink solution of [1] which consists of the square patch of **B2b** patched continuously to the two symmetry breaking solutions $f = \pm\sqrt{c}$, $r = -2c$. On general grounds [26] such a patching induces a matter flux along the hypersurface of patching. This was shown explicitly in [2] for the case under consideration, and it was found that the induced matter flux has to be (anti-)self dual.[21]

We briefly recall that discussion and translate it into the second order formulation in order to pinpoint the source of non-smoothness in the oxidized theory in 3D: The dual field strength f as a function of the non Killing coordinate remains continuous but is not differentiable anymore: in the kink region it is linear according to (3.26), while in the two symmetry breaking vacua it is constant (3.24). Because the condition of vanishing Cotton tensor $C^{\mu\nu} = 0$ (cf. (1.13)) implies a second order differential equation for f a δ-function appears in it generating the induced matter flux. Thus, the eqs. of motion have to be modified to $C^{\mu\nu} = T^{\mu\nu}$, where $T^{\mu\nu}$ is the induced matter flux on the hypersurface of patching. Note that the non-smoothness is difficult to spot in the coordinate system used in eq. (1.22) because the patching occurs in the "asymptotic region" $x = \pm\infty$. However, as the affine distance to this region is finite the solution can be extended and the smooth extension in 2D is nothing but **B2b**.

In general such a patching procedure might destroy the BPS property. However, for the kink solution there are two reasons for optimism: 1. all patches preserve half of the SUSYs and 2. the induced matter flux is (anti-)self dual. In addition it is purely bosonic as the fermionic background is trivial and hence no fermionic matter flux is induced. Although each of these properties alone is not sufficient to guarantee BPS, taken together they are. Technically, the crucial observation is that the conservation equation in presence of matter, (7.4) of [22], reduces to the one in the absence of matter because 1. no fermions are present, 2. the matter component $X^{++}W^{--}$ vanishes as the matter component $W^{--} = 0$ and 3. the remaining matter contribution $X^{--}W^{++}$ vanishes as well because $W^{++} \neq 0$ is valid only on the horizon where $X^{--} = 0$. Thus, $C = 0$ is still the ground state which is necessary[22] for BPS states with vanishing fermions.

Introducing the coordinate $x \in (-\infty, \infty)$ which coincides with X in the kink region $x \in [-\sqrt{c}, \sqrt{c}]$, the following situation is encountered globally (recalling that the horizons are located at $x = \pm\sqrt{c}$ and defining $x_\pm := x \pm \sqrt{c}$, $\theta_{\text{kink}} := \theta(x_+) - \theta(x_-)$ and $\theta(0) := 1/2$)

$$\epsilon_+ = \epsilon_+(x)\theta_{\text{kink}} + \epsilon_+^{(l)}(x)\theta(-x_+) + \epsilon_+^{(r)}(x)\theta(x_-)\,, \tag{3.27}$$

$$\epsilon_- = \epsilon_-(x)\theta_{\text{kink}} + \epsilon_-^{(l)}(x)\theta(-x_+) + \epsilon_-^{(r)}(x)\theta(x_-)\,, \tag{3.28}$$

[21] Whether self- or anti-self dual depends on the patching: one can glue the two symmetry breaking solutions either on the left and right horizon in **B2b** or on the upper and lower one. We will treat only self dual fluxes explicitly but the anti-self dual ones may be obtained by exchanging + indices with − indices in all subsequent considerations.

[22] That it is also sufficient in the present case can be checked easily by considering the restrictions on the SUSY transformation parameters $\epsilon_\pm$ in section 4 of [22]. Note that within each patch $C = 0$ is both necessary and sufficient for BPS provided the fermions vanish, but one needs to check in addition the compatibility of patching.

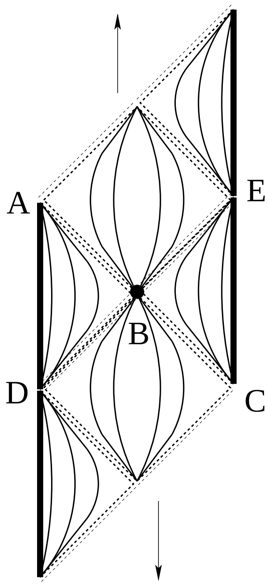

Figure 2: Maximally extended Carter-Penrose diagram for the BPS-kink.

$$f = x\theta_{\text{kink}} - \sqrt{c}\,(\theta(-x_+) - \theta(x_-)) \ , \tag{3.29}$$

$$r = (c - 3x^2)\,\underbrace{\theta_{\text{kink}}}_{\text{kink}} - 2c\big(\underbrace{\theta(-x_+)}_{\text{left AdS}} + \underbrace{\theta(x_-)}_{\text{right AdS}}\big) \ , \tag{3.30}$$

with the following continuity properties of the symmetry parameters

$$\epsilon_\pm(\pm\sqrt{c}) = 0 \, , \tag{3.31}$$

$$\epsilon_\pm(\sqrt{c}) = \epsilon_\pm^{(r)}(\sqrt{c}) \, , \tag{3.32}$$

$$\epsilon_\pm(-\sqrt{c}) = \epsilon_\pm^{(l)}(-\sqrt{c}) \, . \tag{3.33}$$

Additionally, the only non-vanishing component of the energy-momentum tensor induced by patching is given by [2]

$$T^{uu} = \delta(x_-) - \delta(x_+) \, . \tag{3.34}$$

where u is the lightlike coordinate used in (3.21). In a sense, these matter fluxes localized on the extremal horizons can be considered as stabilizing the kink.

In figure 2 the Carter-Penrose diagram for the kink is depicted. Note that the boundaries do not represent a curvature singularity. Time is plotted vertically. The (anti-)self dual shock waves propagate from A (D) to B and from C (E) to B, respectively. The point B is the bifurcation point. The diagram can be continued above and below. It has strip-like topology with singularities at the boundaries. The outer triangle patches are the AdS vacua, the inner square patch surrounded by extremal Killing horizons is the kink region.

Physical instances of the kink. As the model under consideration has no propagating physical modes it is a fair question whether the kink solution is a gauge artifact or a physical entity. At first glance there seems to be no physics, as (1.22) by a *regular* conformal transformation can be mapped to flat space. On the other hand, one can argue on general grounds that the kink solution is as "real" as the (extremal) RN BH, because the latter may be described as well by a (g)PSM with four target space coordinates and a degenerate (bosonic) Poisson tensor of rank 2, the two Casimir functions being related to ADM mass and total charge. In this context one should be rather careful with the definition of mass (for a recent discussion in the framework of 2D dilaton gravity cf. appendix A of [27]). In particular, the conformal frame plays an important role here. The purpose of the rest of this section is to elucidate these issues and to clarify which of the features of the kink solution actually are physical.

In the chosen conformal frame all kink solutions have the same energy, namely the energy of the ground state ($C = 0$). They differ by the value of the U(1) charge c, but qualitatively they are essentially the same (they yield the same Carter-Penrose diagram) apart from the special case $c = 0$. Actually, this is again equivalent to the corresponding properties of the extremal RN BH: all of them have the same BPS mass[23] and the family of extremal solutions may be labelled by the value of the U(1) charge. Note, however, that the charge c is not quantized and thus there is a continuous spectrum of kinks with the same energy.

It is recalled that the 3D scalar field has been set to 1 and thus it is regular everywhere. Therefore, the only singularities in this conformal frame are the ones induced by the metric or the gauge field. As has been shown above, a curvature singularity exists only for $X \to \pm\infty$, which is also the locus where the gauge field diverges. Thus, eventual conformal transformations should be regular everywhere except in the asymptotic regions $X \to \pm\infty$ where they may be singular, in order to avoid the introduction of spurious singularities.

Why is it of interest to study different conformal frames at all? First of all, as the theory exhibits conformal invariance it may be considered as a consistency check. Additionally, any mass definition depends crucially on the selected ground state geometry and the latter may exhibit slightly different behavior in different conformal frames. Moreover, it may well be that some features of the kink encountered above are actually an artifact of the chosen frame and thus it is of interest to separate these from the true physical features.

Appendix C is devoted to a study of conformal transformations with a conformal factor monomial in X. However, all these transformations destroy the kink solution because they are singular at $X = 0$ and thus either a curvature singularity is introduced at $X = 0$ (so there is no interpolating solution available between positive and negative X anymore) or the point $X = 0$ is mapped to infinity (and thus the kink solution may not pass through this point). The singularity of the conformal factor at $X = 0$ is in contradiction with our requirement of regularity and thus one needs at least binomials. Rather than repeating a (somewhat tedious) study analogous to appendix C a short cut will be taken: having in mind the nice features of the AdS ground state (C.10) and the requirement of regularity at $X = 0$ it is easy to find a convenient conformal factor

$$e^Q = \alpha X^2 + \delta\,, \tag{3.35}$$

[23]For the RN BH the BPS mass is defined as $M_{BPS} := M_{ADM} - |Q|$. Thus, by definition all BPS states have vanishing BPS mass, while their ADM mass is given by the charge. Our definition of the Casimir function is adjusted such that C vanishes for BPS states.

with some positive α, δ. Obviously, (3.35) is regular everywhere except for the asymptotic region in accordance with our requirements. The constant δ defines how geometry is rescaled at the origin $X = 0$ and may be set to 1, while α sets the scale for the asymptotically constant curvature (C.11). With $\tilde{X} = \alpha X^3/3 + \delta X$ the line element is transformed to (X is understood to be a function of $\tilde{X}$)[24]

$$\mathrm{d}s^2 = 2\,\mathrm{d}u\,\mathrm{d}\tilde{X} + (\alpha X^2 + \delta)\left(2C + \frac{(X^2 - c)^2}{4}\right)\mathrm{d}u^2\,. \tag{3.36}$$

Note that (3.35) by virtue of (C.2) implies the somewhat unusual bosonic potential $Z = 2\alpha X/(\alpha X^2 + \delta^2)$ which only asymptotically has the "standard form" $Z = \beta/X$ with $\beta = 2$. The bosonic second order action in this frame instead of (1.15) takes the form of (1.12) with $\phi = e^Q = \alpha X^2 + \delta$. The relation between the second order gauge field a and the first order one A up to pure gauge terms now reads

$$a = \frac{X^2}{\int^X e^{Q(X')} X'\,\mathrm{d}X'} A = \frac{X^2}{2}\,\mathrm{d}u\,. \tag{3.37}$$

By construction a is invariant under conformal transformations while A is not. Correspondingly, the dual field strength is related to the dilaton field by

$$f = -Xe^{-Q(X)} = -\frac{X}{\alpha X^2 + \delta}\,. \tag{3.38}$$

One can solve uniquely for X in terms of f noticing that $f \to 0$ implies $X \to 0$.

In the conformal frame implied by (3.35) geometry approaches asymptotically AdS and thus it may be joined continuously to the symmetry breaking constant dilaton vacua if α is chosen as $\alpha = 9/(4c)$. Therefore, the conformal factor depends on the charge c but is independent from the mass C. No patching at the Killing horizons is necessary and consequently no matter fluxes are induced. The solution avoids curvature singularities everywhere. Up to globally regular conformal transformations this conformal frame is unique. Thus, any physical discussion should be based upon this frame.

To summarize, the physical features of the kink solution are 1. the ground state property (they are all solutions of lowest energy), 2. the nonvanishing U(1) charge c, 3. the existence of two extremal Killing horizons in a basic Eddington-Finkelstein patch, 4. the fact that it interpolates between the two symmetry breaking AdS vacua. In the original frame the last point could not be achieved smoothly but only in the presence of (self-dual) matter fluxes. In the frame implied by the conformal factor (3.35) all solutions asymptotically approach AdS_2 and thus the symmetry breaking AdS vacua may be patched continuously and without inducing matter fluxes provided α is tuned to $9/(4c)$. We will comment on this asymptotic behavior in more detail in section 4.2.

3.3 Hawking temperature

As some of the solutions exhibit (Killing) horizons the Hawking effect [28] is of relevance. There are many ways to calculate the Hawking temperature, some of them involving the

[24]Explicitly: $X = W^{-1/3}\delta - W^{1/3}/\alpha$ with $W = \frac{3}{2}\alpha^2\tilde{X}\left(\sqrt{1 + 4\delta^3/(9\alpha\tilde{X}^2)} - 1\right)$

coupling to matter fields, some of them being purely geometrical. Because of its simplicity and since we do not intend to introduce matter explicitly, we will restrict ourselves to a calculation of the geometric Hawking temperature as derived from surface gravity (cf. e.g. [29]).[25] The latter can be calculated by taking the normal derivative $\mathrm{d}/\mathrm{d}X$ of the Killing norm $K(X;C,c)$ given in eq. (3.22), evaluated on one of the Killing horizons $X = X_h$, where X_h is a solution of $K(X_h;C,c) = 0$:

$$T_H = \frac{1}{4\pi}\left|\frac{\mathrm{d}}{\mathrm{d}X}K(X;C,c)\right|_{X=X_h} = \frac{1}{2\pi}\Big|V(X,Y)\Big|_{X=X_h}^{Y=c} = \frac{1}{4\pi}\left|X_h(c-X_h^2)\right| . \tag{3.39}$$

The numerical prefactor in (3.39) can be changed by a redefinition of the Boltzmann constant. It has been chosen in accordance with refs. [30, 12]. The zeros of the Killing norm are given by

$$X_h = \pm\sqrt{c \pm 2\sqrt{2M}} . \tag{3.40}$$

Here it should be recalled that for sake of backward compatibility $C \leq 0$ for positive mass configurations [12]; we have thus defined $M = -C$, thereby fixing an irrelevant scale factor between minus the Casimir and mass to unity. Henceforth $M \geq 0$ will be assumed; the bound is saturated only for the ground state geometries. The Hawking temperature may be presented as

$$T_H = \frac{1}{2\pi}\sqrt{2M}\sqrt{c \pm 2\sqrt{2M}} , \tag{3.41}$$

where the $+$ sign refers to the outer horizons (which exist as long as $c \geq -2\sqrt{2M}$) and the $-$ sign to eventual inner ones (which may exist only for $c \geq 2\sqrt{2M}$). Thus, in the large M limit one obtains $T_H \propto M^{3/4}$, while in the large c limit one gets $T_H \propto \sqrt{cM}$. Remarkably, the specific heat

$$C_s = \frac{\mathrm{d}M}{\mathrm{d}T_H} = \frac{4\pi^2 T_H}{c \pm 3\sqrt{2M}} \tag{3.42}$$

is always positive on outer horizons. It may become negative on inner horizons provided that $3\sqrt{2M} > c > 2\sqrt{2M}$. Somewhat unexpectedly for $c = 3\sqrt{2M}$ the inverse specific heat vanishes on the inner horizon. In order to compare these results with similar well-known ones, it is recalled that for the Schwarzschild BH the specific heat is always negative (cf. e.g. [29]), while for the Witten BH [31] the inverse specific heat vanishes. It is worthwhile mentioning that the Hawking-Page transition between an AdS-Schwarzschild BH and pure AdS filled with radiation [32] occurs for $C_s^{-1} = 0$. Thus, although the thermodynamical analysis of a multi-horizon geometry clearly is more involved, a similar behavior might be expected in our case at $c = 3\sqrt{2M}$, corresponding to a critical temperature of

$$T_{\text{critical}} = \frac{1}{2\pi}\left(\frac{c}{3}\right)^{3/2} . \tag{3.43}$$

In addition to being interesting on its own, the result (3.41) provides a consistency check for our BPS solutions: according to a well-known argument due to Gibbons [25] BPS solutions

[25] If defined in this way Hawking temperature turns out to be independent of the conformal frame. Thus, we may stick to the simple conformal frame where no kinetic term of the dilaton arises (cf. eq. (C.5) in appendix C). For a review on Hawking radiation in $D = 2$ cf. e.g. [30].

JHEP11(2004)021

must have vanishing Hawking temperature. Indeed, for $M = 0$ it vanishes. Incidentally, for $M = c^2/8$ and $c > 0$ there is also a non-BPS solution which nevertheless has vanishing Hawking temperature. It corresponds to the temperature dictated by the extremal inner horizon in the solution **B3** of [2] which additionally exhibits two nonextremal outer horizons and thus cannot be BPS globally.

It could be of interest to check our straightforward analysis by one of the more involved derivations of the Hawking effect, in particular one where matter is present.

4. Higher dimensional perspective

In this section, we would like to analyze the SUSY of the 2D kink solution, but from a 3D viewpoint. Finally, we will also provide a connection with M-theory membranes, that was basically spelled out in [33]. We will also comment on AdS/CFT aspects of the 11D SUGRA solution.

Let us start by studying the kink geometry in 3D SUGRA. In this case the line element is

$$\mathrm{d}s^2 = e^{2g(x)}(-\,\mathrm{d}x^2 - \mathrm{d}r^2 - 2A(x)\,\mathrm{d}t\,\mathrm{d}r)\,. \tag{4.1}$$

The coordinates t, x are equivalent to the corresponding ones in $D = 2$, while r denotes the Killing direction that has been eliminated by Kaluza-Klein reduction. We take this form of the conformal factor, because we are interested in solutions that are basically 2D and because we will consider static solutions. The presence of the conformal factor $e^{2g(x)}$ will be important in the following. With this metric, we can associate a set of Vielbeine

$$e^0 = e^{g(x)}A(x)\,\mathrm{d}t\,, \qquad e^1 = e^{g(x)}\,\mathrm{d}x\,, \qquad e^2 = e^{g(x)}(\mathrm{d}r + A(x)\,\mathrm{d}t)\,, \tag{4.2}$$

which yields the spin connection ω_μ^{AB}

$$\omega_r^{01} = \omega_x^{02} = -\frac{A'(x)}{2A(x)}\,, \qquad \omega_t^{01} = \omega_t^{21} = \frac{A'(x)}{2} + A(x)g'(x)\,, \qquad \omega_r^{21} = g'\,. \tag{4.3}$$

Now, it will be assumed that the spinor ϵ depends only on the coordinate x. Consequently, the 3D SUSY transformations [9]

$$\delta\psi_\mu = \partial_\mu\epsilon + \frac{1}{2}\omega_\mu^{AB}\sigma_{AB}\epsilon \tag{4.4}$$

with $\sigma_{AB} := \Gamma_A\Gamma_B$ imply

$$\delta\psi_t = 0 \to \partial_t\epsilon + \left(\frac{A'}{2} + Ag'\right)(\sigma_{01} + \sigma_{21})\epsilon = 0 \tag{4.5}$$

$$\delta\psi_x = 0 \to \partial_x\epsilon - \frac{A'}{2A}\sigma_{02}\epsilon = 0 \tag{4.6}$$

$$\delta\psi_r = 0 \to \partial_r\epsilon + \left(\frac{A'}{2A}\sigma_{10} + g'\sigma_{21}\right)\epsilon = 0\,. \tag{4.7}$$

To impose that (part of) the SUSY is preserved, then we have to solve,

$$\left(\frac{A'}{2} + Ag'\right)\sigma_{01}(1 + \sigma_{20})\epsilon = 0 \tag{4.8}$$

$$\partial_x \epsilon - \frac{A'}{2A}\sigma_{02}\epsilon = 0 \tag{4.9}$$

$$\left(-\frac{A'}{2A}\sigma_{02} + g'\sigma_{21}\right)\epsilon = 0\,. \tag{4.10}$$

We can solve (4.8) if[26]

$$\sigma_{20}\epsilon = -\epsilon\,, \tag{4.11}$$

plugging into (4.9) gives

$$\epsilon(x) = e^{-\int \frac{A'}{2A}\,\mathrm{d}x}\epsilon(0)\,. \tag{4.12}$$

and in solving (4.10)

$$g' = -\frac{A'}{2A}\,. \tag{4.13}$$

So, this provides an eq. for the conformal factor, in terms of the function $A(x)$.

There are important points to be noticed here. First, the fact that the conformal factor $g(x)$ is nonzero, allows for a fraction of SUSY to be preserved. Indeed, had we taken a $g(x) = 0$, this would be reflected in the spin connection (4.3) and eqs. (4.8), (4.10) would not have solution. Second, the conformal factor is determined in terms of the function $A(x)$. Third, the form of the spinor and the eq. (4.5) are analogous to eqs. (3.3), (3.4) in the 2D analysis.

The reader might wonder why (in a conformally invariant theory) the conformal factor is determined by eq. (4.13). Indeed, one may think that *any* conformal factor will be the same. But preservation of some fraction of SUSY imposes a constraint. Indeed, while the projection (4.11) is breaking part of the SUSY, it is also spoiling the superconformal invariance, thus leading to a unique conformal factor. Summarizing, all possible conformal factors will be solutions of the eqs. of motion $C^{\mu\nu} = 0$, but only a subset of those solutions is supersymmetric. This subset is described (if our trial line element is of the 'kink' form) by metrics like

$$\mathrm{d}s^2 = \frac{1}{A(x)}(-\,\mathrm{d}x^2 - 2A(x)\,\mathrm{d}t\,\mathrm{d}r - \mathrm{d}r^2)\,, \tag{4.14}$$

with scalar curvature vanishing for arbitrary A. Notice also, that it is possible that more general supersymmetric solutions exist, where the conformal factor depends on more than one coordinate, but still preserving the same trial for the line element. Finally, it should be stressed that the conformal factor in (4.14) becomes singular at Killing horizons of the 2D geometry $A(x) = 0$. Thus, (4.14) may be mapped to Minkowski space by a globally *regular* conformal transformation in 2D, where "gobally" refers to a region where $A \neq 0$. Therefore, in 3D the only SUSY solution is globally conformal to Minkowski space.

For sake of completeness we provide the general bosonic result for the 3D metric. It follows directly from the *Ansatz* (1.4) with the gauge field, expressed in a convenient

[26]Note that (4.11) is a direct consequence of (4.8), unless (4.13) holds. Since this is actually the case we nevertheless *impose* (4.11) as an independent constraint in order to be able to solve (4.9).

gauge,[27]

$$a = \frac{X^2 - c}{2}\, \mathrm{d}u\,. \tag{4.15}$$

and the 2D line element (3.21), (3.22):

$$\mathrm{d}s^2 = 2C\, \mathrm{d}u^2 + 2\, \mathrm{d}u\, \mathrm{d}X - (X^2 - c)\, \mathrm{d}u\, \mathrm{d}r - \mathrm{d}r^2\,. \tag{4.16}$$

Only for BPS states the $\mathrm{d}u^2$ term vanishes. The appearance of the prepotential (2.15) in (4.15) and (4.16) should be noted. The 3D scalar curvature

$$R = c - \frac{5X^2}{2}\,, \tag{4.17}$$

is independent from C in this particular conformal frame. For consistency it can be checked easily that the Cotton tensor derived from (4.16) indeed vanishes. A coordinate transformation to conformal gauge is possible in principle but somewhat tedious; necessarily it has singularities at values of the dilaton field given by (3.40). Therefore, in 3D, much like in 2D, conformal gauge need not be a convenient choice for calculations and especially not for the discussion of the global structure. For the simple case $C = 0$ by a gauge transformation $a \to a + 2\, \mathrm{d}X/(X^2 - c)$ and a coordinate redefinition $\mathrm{d}t = -c\, \mathrm{d}u/2 - 2c\, \mathrm{d}X/(X^2 - c)^2$, $\mathrm{d}x = -2\, \mathrm{d}X/(X^2 - c)$ the line element for $X \in (-\sqrt{c}, \sqrt{c})$ may be presented in a way coinciding with (4.56) of [1],

$$\mathrm{d}s^2\big|_{C=0} = -\, \mathrm{d}x^2 - \frac{2\, \mathrm{d}t\, \mathrm{d}r}{\cosh^2 \frac{\sqrt{c}x}{2}} - \mathrm{d}r^2\,. \tag{4.18}$$

An explicit transformation of (4.18) to conformal gauge can be found in ref. [34]. We will comment in detail on the asymptotic behavior in section 4.2.

4.1 M theory perspective

Let us now consider the same solution from a higher dimensional perspective. We will try to summarize a part of the paper [33], that deals with a system in 4D U(1) gauged SUGRA that has the same eqs. of motion that the kink solves. Then, we offer an M-theory interpretation of the kink by using the lifting prescription given in [35]. The general idea is that a particular configuration in 4D SUGRA, gives place to a set of eqs. of motion that are exactly the ones derived from our 2D action (2.17). As is well known, the bosonic part of (2.17) can be derived from reduction of the action (1.1) as is shown in ref. [1]. So, in this section we want to connect this three systems. Besides, the outcome of [33] is that the configuration is supersymmetric, the same we have shown to happen in our 2D system and from a 3D perspective.

Let us start by considering $\mathcal{N} = 2$ 4D U(1) gauged SUGRA; the bosonic part of the lagrangian reads

$$L = \sqrt{g}\left[R - F_{\mu\nu}^2 + \frac{6}{l^2}\right]. \tag{4.19}$$

[27] In other gauges the constant c may be replaced by an arbitrary function of u and an additional exact term $f(X)\, \mathrm{d}X$ may be present in a.

JHEP11(2004)021

The field content of this SUGRA is given by the metric, a U(1) gauge field A_μ and a complex gravitino ψ_μ. The supersymmetry transformations look

$$\delta e_\mu^a = \Re[\bar{\epsilon}\gamma^a\psi_\mu]\,, \qquad \delta A_\mu = \Im[\bar{\epsilon}\psi_\mu]\,, \qquad \delta\psi_\mu = \hat{D}_\mu\epsilon\,, \tag{4.20}$$

where

$$\hat{D}_\mu\epsilon = \left[\nabla_\mu(\omega) - \frac{i}{l}A_\mu + \frac{1}{2l}\gamma_\mu + \frac{i}{4}F_{ab}\gamma^{ab}\gamma_\mu\right]\epsilon \tag{4.21}$$

and γ_{ab} is the usual antisymmetrized product of gamma matrices.

In [33] it was shown that a certain class of BPS solutions can be represented as (in this section we use the signature $(-1,1,1,1)$ in accordance with [33]),

$$\mathrm{d}s^2 = -f(r,x)^2(\mathrm{d}t - a(r,x)\,\mathrm{d}z)^2 + \frac{1}{f(r,x)^2}\,\mathrm{d}r^2 + \frac{g(r,x)^2}{1+\delta(x)}\left(\frac{(1+\delta(x))^2\,\mathrm{d}z^2}{\cosh^4(x/2)} + \mathrm{d}x^2\right), \tag{4.22}$$

with gauge field,

$$A = s(r,x)\left(\left(r^2 + \frac{1}{4}\right)\mathrm{d}t + \frac{\delta_0}{4}\,\mathrm{d}z\right). \tag{4.23}$$

The coordinates t, x, r essentially correspond to the 3D coordinates used in (4.1) and

$$f(r,x) = \frac{r^2+1/4}{g(r,x)}\,, \quad a(r,x) = \frac{1+\delta(x)}{4(r^2+1/4)\cosh^4(x/2)} - \frac{1}{\cosh^2(x/2)} \tag{4.24}$$

$$s(r,x) = \frac{\tanh(x/2)}{2g^2(r,x)}\,, \quad g(r,x) = \sqrt{r^2 + 1/4\ \tanh^2(x/2)}\,, \tag{4.25}$$

$$\delta(x) = \delta_0\cosh^4\left(\frac{x}{2}\right), \quad \delta_0 = \frac{8C}{c^2}\,. \tag{4.26}$$

Note that the scalar curvature is a negative constant globally for all solutions. In the last equation C and c refer to the Casimir functions (3.18) and (3.19) and accordingly the kink solution is recovered for $\delta_0 = 0$ (recall the definition of C in (3.18) and the subsequent discussion). In the latter case the eqs. of motion derived from (4.19), are satisfied,

$$R_{\mu\nu} = 2\left(F_{\mu\rho}F^\rho{}_\nu - \frac{1}{4}g_{\mu\nu}F^2\right) - 3g_{\mu\nu} \tag{4.27}$$

with the gauge field strength

$$F = \alpha e^0\wedge e^1 + \beta e^0\wedge e^2 + \alpha e^1\wedge e^3 + \beta e^2\wedge e^3\,, \tag{4.28}$$

where[28]

$$\alpha = \gamma r\tanh\left(\frac{x}{2}\right), \quad \beta = -\gamma\left(r^2 - \tanh^2(x/2)/4\right), \quad \gamma = \frac{1}{(2g^2\cosh(x/2))^2}\,. \tag{4.29}$$

To obtain (4.28) the tetrad

$$e^0 = f(\mathrm{d}t - a\,\mathrm{d}z)\,, \quad e^1 = \frac{\mathrm{d}r}{f}\,, \quad e^2 = g\,\mathrm{d}x\,, \quad e^3 = \frac{g\,\mathrm{d}z}{\cosh^2(x/2)} \tag{4.30}$$

[28] As from now on we set $\delta_0 = 0$, the (useful) relations $fg = (r^2+1/4)$ and $g = -fa\cosh^2(x/2)$ hold.

has been employed and $F = \mathrm{d}A$ has been used with

$$\mathrm{d}A = \left(\frac{\partial s}{\partial r}\left(r^2+\frac{1}{4}\right)+2rs\right)\mathrm{d}r\wedge\mathrm{d}t+\frac{\partial s}{\partial x}\left(r^2+\frac{1}{4}\right)\mathrm{d}x\wedge\mathrm{d}t\,. \tag{4.31}$$

This implies an electric field $E = (\alpha, \beta, 0)$ and a magnetic field $B = (\beta, -\alpha, 0)$ (the entries refer to $1, 2, 3$-component, resp.). We can get $*F$ either by applying Hodge to (4.28) or by replacing $\alpha \to \beta$ and $\beta \to -\alpha$. Note that $F \wedge *F = 0$ but F is not proportional to $*F$, thus we have neither self duality nor anti-self duality. The following physical consequences should be noted: both scalar invariants built from F vanish: $E^2 - B^2 = 0$ and $E \cdot B = 0$. The energy-density turns out as $H = \frac{1}{2}(E^2+B^2) = \alpha^2+\beta^2 = \gamma^2$ (for large x energy behaves as $H \approx e^{-2x}/(r^2+1/4)^2$, for large r it reads $H \approx 1/(2r\cosh{(x/2)})^4$). The Poynting vector $E\times B = -H(0,0,1)$ shows into negative 3-direction. This is the behavior expected from an ordinary electro-magnetic wave in curved space. Note that the metric (4.22) is stationary but not static, a property which also holds for the field strength. One could call this field configuration a soliton consisting of photons kept together by gravity: it is a regular field configuration and energy is bounded for all values of x, r; also, energy falls off for large x, r. Moreover, the nonvanishing flux and its fall-off behavior explain why the Ricci tensor is nontrivial but tends to the one of AdS asymptotically ($x \to \infty$).

For very large values of the coordinate x, the metric looks like

$$\mathrm{d}s^2 = \left(r^2+\frac{1}{4}\right)(-\,\mathrm{d}t^2+\mathrm{d}x^2+e^{-x}\,\mathrm{d}t\,\mathrm{d}z)+\frac{\mathrm{d}r^2}{(r^2+\frac{1}{4})} \tag{4.32}$$

whilst the gauge field (4.23) is pure gauge. For $x \to \infty$ this space is AdS_4. We will now discuss the asymptotics in more detail in order to decide whether we have $AAdS$ according to the definition given, for example, in ref. [36].

4.2 AdS asymptotics in $D = 2, 3, 4$

As there are several subtleties involved in the discussion of the asymptotic behavior it is worthwhile to collect them at this point. Because most of them arise already in the much simpler framework of the 2D solutions we will study them first.

We recall that the Riemann tensor has $D^2(D^2-1)/12$ independent components. Consequently, in $D = 2$ the Riemann tensor may be expressed solely in terms of the scalar curvature. Thus, it is sufficient in $D = 2$ to study the asymptotic behavior of the latter. Now we encounter the first subtlety:

Definition of asymptotic region. The line element (1.22) suggests that the asymptotic region is located at $x \to \pm\infty$. One can then check that, according to the definition of $AAdS$ provided in ref. [36], the kink solution is $AAdS_2$. However, we have seen that $x \to \pm\infty$ does not correspond to the asymptotic region but rather to a Killing horizon. The asymptotic region of the analytically extended metric (3.21) is reached for $X \to \pm\infty$. One can deduce immediately from (3.23) that the metric is not $AAdS$. At least, not in the conformal frame employed there. This brings us to the second subtlety:

Definition of the conformal frame. Classical conformal invariance gives us the freedom to choose any non-singular conformal factor which we deem to be convenient. Actually, we may allow for singularities in the asymptotic region in order to get rid of the curvature singularity, but for finite X the conformal factor must not vanish or diverge. Exploiting this freedom via eq. (3.35) we have shown that it is possible to bring the metric to a form which asymptotically approaches AdS, cf. eqs. (C.9)–(C.11). Thus, in a naive sense all our solutions asymptotically are AdS. However, there is another subtlety which appears to spoil the *AAdS* property:

Next to leading order terms. It is mentioned in ref. [36] that the Riemann tensor not only has to approach the one of AdS space in the asymptotic region, but also that the next to leading order (NLO) terms should have a certain fall-off behavior and in particular an ordinary Taylor series in powers of the "defining function" should emerge. Let us study this in detail (for sake of definiteness we will restrict ourselves to $X \to \infty$). The asymptotic Killing norm in the frame implied by (3.35) reads $K = (\alpha/4)X^6 + \mathcal{O}(X^4)$. With $\tilde{X} = (\alpha/3)X^3 + \delta X$ (one over the "defining function" in the parlance of [36]) the asymptotic line element in conformal gauge ($\mathrm{d}s^2_{\text{flat}} = 2\,\mathrm{d}u\,\mathrm{d}v$ with $\mathrm{d}v = \mathrm{d}u/2 + \mathrm{d}\tilde{X}/K(\tilde{X})$) reads (note that $\alpha > 0$)

$$\mathrm{d}s^2_{\text{asy}} = \frac{9}{4\alpha}\tilde{X}^2 \left(1 + \mathcal{O}(\tilde{X}^{-2/3})\right) \mathrm{d}s^2_{\text{flat}}\,. \tag{4.33}$$

As required, it has a second order pole in $1/\tilde{X}$ at the boundary. However, the NLO terms of (4.33) contain third roots and do not allow for an ordinary power series around $\tilde{X} = \pm\infty$. The only exception is the trivial case $C = c = 0$ implying not only $AAdS_2$ but AdS_2. Thus, although near the AdS boundaries curvature is given by $r = -9/(2\alpha) + \mathcal{O}(\tilde{X}^{-2/3})$ spacetime is not *AAdS* in the technical sense. If $\alpha = 9/(4c)$ is chosen for matching continuously to the constant dilaton vacua (as explained below (3.35)) curvature at the boundary is given by $r = -2c$, in accordance with (1.18)-B. It may well be that for some applications the appearance of third roots in the NLO terms is not problematic, but it is emphasized again that in the strict sense none of our solutions is *AAdS*, despite of the correct asymptotic limit of the Riemann tensor and the line element.

Oxidation to $D = 3$. In fact, most of the previous discussion "oxidizes" to $D = 3$. For a negative result it is sufficient to show that the scalar curvature spoils the *AAdS* property. A one-to-one repetition of the line of reasoning given above, including all caveats mentioned, leads to the conclusion that none of our solutions, eq. (4.34) below, is $AAdS_3$ in the strict sense because of the NLO terms. What about being asymptotically AdS in the weaker sense explained above? We present a general argument why the metric can be brought into a form which asymptotically approaches AdS_3: because the Cotton tensor (1.13) vanishes globally the line element (4.16) is conformally flat. But since AdS_3 is conformally flat as well it is possible by two consecutive conformal transformations to bring the line element to AdS_3. However, the conformal factor will be singular at Killing horizons of (4.16). Thus, by excluding singularities in the conformal factor everywhere except in the asymptotic region it is only possible to transform (4.16) to a form which asymptotically, i.e., for $X \to \pm\infty$, approaches AdS_3. Therefore it is tempting to use the line element

$$\mathrm{d}s^2 = e^Q \left(2C\,\mathrm{d}u^2 + 2\,\mathrm{d}u\,\mathrm{d}X - (X^2 - c)\,\mathrm{d}u\,\mathrm{d}r - \mathrm{d}r^2\right), \tag{4.34}$$

with Q as defined in (3.35). At first glance prospects look promising: in $D = 3$ the Riemann tensor is fully determined by specifying the trace free Ricci tensor and the curvature scalar, and the latter tends to a negative constant asymptotically. Additionally, in 3D the Cotton tensor $C^{\mu\nu}$ has as many algebraically independent components as the trace free Ricci tensor, namely 5 [37], and the eqs. of motion imply $C^{\mu\nu} = 0$. However, $C^{\mu\nu}$ contains one additional derivative as compared to the trace free Ricci tensor and thus less information. Therefore, it is conceivable that some integration constants are present in the trace free Ricci tensor while $C^{\mu\nu} = 0$. Indeed, this turns out to be the case for the line element (4.34). Consequently, our 3D solutions (4.34) are not even asymptotically AdS in a weak sense. It seems likely that the conformal factor which brings (4.34) to a form which is asymptotically AdS_3 has to depend not only on the dilaton X, but also on the other two coordinates.

Consequences for $D = 4$. We have mentioned in the previous section that the 4D analogue of the BPS solution asymptotically approaches AdS_4, so it could well be that it is $AAdS_4$. However, again most of the caveats mentioned above apply: In particular, the "asymptotic region" $x \to \pm\infty$ has been employed, but since neither in $D = 2$ nor in $D = 3$ this corresponds to the true asymptotic region it remains an open question whether this is the case in $D = 4$. Moreover, while the limit $x \to \pm\infty$ of the Riemann tensor undoubtedly is consistent with AdS_4 the NLO terms have not been checked. Finally, because the scalar curvature is constant for all solutions (4.22) and because the Weyl tensor $C^{\mu}{}_{\nu\sigma\tau}$ vanishes[29] in the limit $r \to \infty$ also the non-BPS solutions exhibit nice asymptotic features: they approach AdS_4 in that limit and the gauge field becomes pure gauge. It could be worthwhile to extend our analysis of the asymptotic behavior in $D = 4$, in particular, to resolve what is actually meant by "asymptotic". This will be left for future work.

4.3 Lifting to M theory

When lifting this 4D solution to M theory, we use the prescription given in [35]. We define the quantities (note that $\Sigma_i \mu_i^2 = 1$),

$$\mu_1 = \sin\theta\,, \quad \mu_2 = \cos\theta\sin\varphi\,, \quad \mu_3 = \cos\theta\cos\varphi\sin\psi\,, \quad \mu_4 = \cos\theta\cos\varphi\cos\psi\,. \tag{4.35}$$

So that the seven sphere can be written as

$$\mathrm{d}\Omega_7^2 = \sum_i [(\mathrm{d}\mu_i)^2 + \mu_i^2\,\mathrm{d}\phi_i^2]\,, \qquad i = 1, \dots, 4\,. \tag{4.36}$$

The 11D metric and four form read (the star denotes the 4D Hodge star),

$$\mathrm{d}s_{11}^2 = \mathrm{d}s_4^2 + 4\sum_i \left[(\mathrm{d}\mu_i)^2 + \mu_i^2\left(\mathrm{d}\phi_i - \frac{1}{2}A_t\,\mathrm{d}t\right)^2\right] \tag{4.37}$$

[29]It is emphasized that this is not true for the index position $C_{\mu\nu\sigma\tau}$. However, the index position used in the main text appears to be preferable because only then the Weyl tensor is conformally invariant.

$$F_4 = -3\epsilon_4 - \sum_i [\mathrm{d}(\mu_i^2) \wedge (\mathrm{d}\phi_i + A_t\,\mathrm{d}t)] \wedge {}^*F_2\,. \tag{4.38}$$

So, we see that for large values of the coordinate x the field strength F_2 in (4.28) vanishes and the configuration looks like M2 branes. This asymptotic metric, of the form $AdS_4 \times S^7$ is being deformed by the presence of the fibration between the four dimensional part $\mathrm{d}s_4^2$ and the internal space, a deformed S^7. To compensate for this change and satisfy the Einstein eqs., the four form field strength gets the second factor in eq. (4.38).

The fact that the metric may be $AAdS$ opens the possibility of studying AdS/CFT aspects of the 11D solution. So, this solution is representing a dual to a 3D conformal theory, that is being deformed by the insertion of an operator or a VEV in the UV, this insertion or VEV also breaks part of the SUSY. We could compute correlation functions by following the prescription in [36]. In order to explore the above mentioned issues, one should first write the metric in the form indicated in ref. [36]. It appears to be difficult to find the explicit transformation, though it should be possible in principle.[30] We leave the AdS/CFT analysis for a future work.

To end this section it would be remark to note the following points. On the one hand it should be interesting to make a full mapping between the 4D gauged SUGRA in eq. (4.19) and the 2D SUGRA in (2.17). If this mapping is possible for any solution of (2.17), then, the matter fluxes that stabilize the kink as discussed in section 3.2 could be identified with the gauge fields in the 4D context and by lifting, with the M theory four form field strength. On the other hand, the mass formula discussed in section 3.2 should be related to the ADM mass of the 4D or 11D solution. There should be a relation between the Hawking temperature discussed in section 3.3 from the 2D viewpoint and some observable in 4D gauged SUGRA or M theory, which will be addressed in a future work as well.

5. Discussion and conclusions

One of our main results was the construction of a supersymmetric extension of the dimensionally reduced gravitational Chern-Simons term

$$\mathcal{S}_{\mathrm{SUCS}} = -\frac{1}{8\pi^2}\int \mathrm{d}^2x\sqrt{-g}\Big(\tilde{r}F + F^3 + \Sigma^2 - F^2\Delta\Big) \tag{5.1}$$

with SUSY transformations

$$\delta e_m{}^a = -2i(\varepsilon\gamma^a\psi_m)\,, \tag{5.2}$$

$$\delta a_m = -2\varepsilon\gamma_*\psi_m\,, \tag{5.3}$$

$$\delta\psi_{m\alpha} = -\hat{D}_m\varepsilon_\alpha\,. \tag{5.4}$$

We have also shown that this model is not equivalent to the reduction of the 3D SUCS, as the fermionic transformations and potential differ. We will address this model in future work.

[30] Mauro Brigante has found a change of variables that leaves the metric (4.32) in the form $g = \phi(t,x,r,z)\eta$ where η is the flat metric.

We have recalled how to obtain all classical solutions locally and globally, namely by casting (5.1) into first order form and exploiting some of the features of graded Poisson Sigma Models. By the use of gPSM techniques global properties of the kink solution including its BPS characteristics have been discussed rigorously. These results were compared with the higher dimensional perspective.

The thermodynamical behavior of SUCS solutions is quite non-trivial. Here we summarize briefly the various phases, supposing $c > 0$ (in brackets the global solution in the notation of ref. [2] is provided):[31]

- $M < 0$: no horizon (**B0**)
- $M = 0$: two extremal horizons, BPS solution, Hawking temperature vanishes, entropy[32] is proportional to $\sqrt{c}$ (**B2b**)
- $0 < \sqrt{2M} < c/3$: four horizons, inner ones: positive specific heat (**B4**)
- $\sqrt{2M} = c/3$: four horizons, Hawking-Page like transition on inner horizons: specific heat diverges (**B4**)
- $c/3 < \sqrt{2M} < c/2$: four horizons, inner horizons: negative specific heat (**B4**)
- $\sqrt{2M} = c/2$: two non-extremal and one extremal horizon, Hawking temperature and entropy vanish on inner (extremal) horizon (**B3**)
- $\sqrt{2M} > c/2$: two horizons (**B2a**).

The specific heat on outer horizons is always positive. It is emphasized that the simple thermodynamical relations presented in this paper are but the first step to a thorough analysis. In particular, the presence of several horizons with different surface gravities has to be taken into account properly. Also, a better (microscopical) understanding of the entropy — e.g. for the BPS solutions where it grows like the square-root of the U(1) charge — would be desirable. Finally, the Hawking-Page like transition implied by the pole in (3.42) should be studied in more detail. Especially the last point could be rewarding to pursue due to its relevance for AdS/CFT (cf. section 3.2 in ref. [39] and section 2.3 in ref. [40]).

In this context we have addressed the asymptotic behavior of the metric and the Riemann tensor in $D = 2, 3$ and 4 thereby revealing promising features: in a convenient conformal frame all solutions in $D = 2$ and in $D = 3$ asymptotically tend to AdS. However, the next to leading order terms contain third roots which seem to spoil $AAdS$ in the strict sense [36]. Further subtleties have been discussed and their resolution in $D = 4$ remains as an open task for future work. In particular, it has to be clarified what is actually the asymptotic region in $D = 4$ and in $D = 11$ (obtained by lifting the 4D solutions to M theory).

[31] The case $c \leq 0$ can be studied as well, but it is less interesting. The only geometries arising are **B0**, **B1a**, **B1b** and **B2a**, i.e., at most two horizons are possible.

[32] Entropy may be calculated by various methods [38] and in our convention reads $S = 2\pi|X_h|$, where X_h is the value of the dilaton field at a horizon given by (3.40).

JHEP11(2004)021

Let us finally address generalizations of our theory that include coupling to matter. In general, the theory ceases to be a topological one and allows for physical scattering processes. Nevertheless, an exact path integral quantization of geometry, auxiliary fields and ghosts may be performed, thus providing a generating functional for Green functions depending solely on the matter fields and on external sources [41]. It could be of interest to apply the general results in that ref. to the present case and to study scattering in various phases of the model, starting for simplicity with $M = 0$. However, dimensional reduction makes it natural to assume a non-trivial coupling of matter to the dilaton field. Though it has been shown recently that the basic properties of the quantization procedure in [41] are retained together with non-minimal coupling [42], this case has not yet been worked out in detail. Moreover, non-trivial couplings of the matter fields to the new scalar field Y could yield an even richer structure which will be worthwhile to study.

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A. Notations and conventions

The conventions used in the first order formulation in section 2 are identical to [20], where additional explanations can be found.

Indices chosen from the Latin alphabet are commuting (lower case) or generic (upper case), Greek indices are anti-commuting. Holonomic coordinates are labeled by M, N, O etc., anholonomic ones by A, B, C etc., whereas I, J, K etc. are general indices of the gPSM. The index ϕ is used to indicate the dilaton component of the gPSM fields (note that in the main text we have used X instead for sake of compatibility with the bosonic literature, while here we use ϕ for sake of compatibility with the SUSY literature):

$$X^\phi = \phi \qquad A_\phi = \omega \tag{A.1}$$

$$X^Y = Y \qquad A_Y = A\,. \tag{A.2}$$

The summation convention is always $NW \to SE$, e.g. for a fermion χ: $\chi^2 = \chi^\alpha \chi_\alpha$. Our conventions are arranged in such a way that almost every bosonic expression is transformed trivially to the graded case when using this summation convention and replacing commuting indices by general ones. This is possible together with exterior derivatives acting *from the right*, only. Thus the graded Leibniz rule is given by

$$\mathrm{d}\,(AB) = A\mathrm{d}B + (-1)^B\,(\mathrm{d}A)B\,. \tag{A.3}$$

JHEP11(2004)021

In terms of anholonomic indices the metric and the symplectic 2×2 tensor are defined as

$$\eta_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \epsilon_{ab} = -\epsilon^{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad \epsilon_{\alpha\beta} = \epsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{A.4}$$

The metric in terms of holonomic indices is obtained by $g_{mn} = e_n^b e_m^a \eta_{ab}$ and for the determinant the standard expression $e = \det e_m^a = \sqrt{-\det g_{mn}}$ is used. The volume form reads $\epsilon = \frac{1}{2}\epsilon^{ab} e_b \wedge e_a$; by definition $*\epsilon = 1$.

The γ-matrices are used in a chiral representation:

$$\gamma^0{}_\alpha{}^\beta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \gamma^1{}_\alpha{}^\beta = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \gamma_{*\alpha}{}^\beta = (\gamma^1\gamma^0)_\alpha{}^\beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.5}$$

Covariant derivatives of anholonomic indices with respect to the geometric variables $e_a = dx^m e_{am}$ and $\psi_\alpha = dx^m \psi_{\alpha m}$ include the 2D spin-connection one form $\omega^{ab} = \omega\epsilon^{ab}$. When acting on lower indices the explicit expressions read ($\frac{1}{2}\gamma_*$ is the generator of Lorentz transformations in spinor space):

$$(De)_a = de_a + \omega\epsilon_a{}^b e_b \qquad (D\psi)_\alpha = d\psi_\alpha - \frac{1}{2}\omega\gamma_{*\alpha}{}^\beta\psi_\beta\,. \tag{A.6}$$

Light-cone components are very convenient. As we work with spinors in a chiral representation we can use

$$\chi^\alpha = (\chi^+, \chi^-)\,, \qquad \chi_\alpha = \begin{pmatrix} \chi_+ \\ \chi_- \end{pmatrix}. \tag{A.7}$$

For Majorana spinors upper and lower chiral components are related by $\chi^+ = \chi_-$, $\chi^- = -\chi_+$, $\chi^2 = \chi^\alpha\chi_\alpha = 2\chi_-\chi_+$. Vectors in light-cone coordinates are given by

$$v^{++} = \frac{i}{\sqrt{2}}(v^0 + v^1)\,, \qquad v^{--} = \frac{-i}{\sqrt{2}}(v^0 - v^1)\,. \tag{A.8}$$

The additional factor i in (A.8) permits a direct identification of the light-cone components with the components of the spin-tensor $v^{\alpha\beta} = \frac{i}{\sqrt{2}} v^c \gamma_c^{\alpha\beta}$. This implies that $\eta_{++|--} = \eta_{--|++} = 1$ and $\epsilon_{--|++} = -\epsilon_{++|--} = 1$. The γ-matrices in light-cone coordinates become

$$(\gamma^{++})_\alpha{}^\beta = \sqrt{2}i \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \qquad (\gamma^{--})_\alpha{}^\beta = -\sqrt{2}i \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}. \tag{A.9}$$

B. First order formulation of SUCS

For details on notations appendix A may be consulted. Note that the dilaton is denoted by ϕ in the appendices and not by X for sake of backward compatibility with the literature on dilaton SUGRA. With the prepotential (2.15) the explicit expressions for the Poisson tensor are [19]:

$$P^{a\phi} = X^b \epsilon_b{}^a \tag{B.1}$$

$$
\begin{aligned}
P^{\alpha\phi} &= -\frac{1}{2}\chi^\beta \gamma_{*\beta}{}^\alpha && \text{(B.2)}\\
P^{ab} &= \epsilon^{ab}\Big(\frac{1}{2}(\phi Y - \phi^3) + \frac{1}{8}\chi^2\Big) && \text{(B.3)}\\
P^{\alpha b} &= -\frac{i}{2}\phi(\chi\gamma^b)^\alpha && \text{(B.4)}\\
P^{\alpha\beta} &= -2iX^c\gamma_c^{\alpha\beta} + (\phi^2 - Y)\gamma_*{}^{\alpha\beta} && \text{(B.5)}\\
P^{YI} &\equiv 0\,. && \text{(B.6)}
\end{aligned}
$$

It should be noted that it does not have full rank. The dimension of the kernel is two and thus two Casimir functions[33] exist, related to conserved mass and charge.

The first-order action (2.11) becomes

$$
\begin{aligned}
\mathcal{S}_{\text{FO}} = \int_{\mathcal{M}_2}\Big(\phi\,\mathrm{d}\omega + Y\,\mathrm{d}A + X^a De_a + \chi^\alpha D\psi_\alpha + \epsilon\Big(\frac{1}{2}(\phi Y - \phi^3) + \frac{1}{8}\chi^2\Big) - && \text{(B.7)}\\
- \frac{i}{2}\phi(\chi\gamma^a e_a\psi) + iX^a(\psi\gamma_a\psi) - \frac{1}{2}(\phi^2 - Y)(\psi\gamma_*\psi)\Big)\,. && \text{(B.8)}
\end{aligned}
$$

he eqs. of motion

$$
\begin{aligned}
\mathrm{d}X^I + P^{IJ}A_J &= 0\,, && \text{(B.9)}\\
\mathrm{d}A_I + \frac{1}{2}(\partial_I P^{JK})A_K \wedge A_J &= 0\,, && \text{(B.10)}
\end{aligned}
$$

can be found explicitly in the main text, (3.10)–(3.17). The change of notation between appendix and main text should be noted, e.g. the dilaton field ϕ is denoted by X in the main text; other changes are summarized in footnote 34.

Elimination of X^a and bosonic torsion yields (eq. (2.38) in [19])

$$
\begin{aligned}
\mathcal{S}_{\text{SO}} = \int \mathrm{d}^2x e\Big(\frac{1}{2}\tilde{R}\phi + (\chi\tilde{\sigma}) - \epsilon^{mn}(Y\partial_n A_m) + \frac{1}{2}(\phi Y - \phi^3) + && \text{(B.11)}\\
+ \frac{1}{8}\chi^2 + \frac{i}{2}\phi\epsilon^{mn}(\chi\gamma_n\psi_m) + \frac{1}{2}(\phi^2 - Y)\epsilon^{mn}(\psi_n\gamma_*\psi_m)\Big) && \text{(B.12)}
\end{aligned}
$$

with the SUSY covariant scalar curvature and its supersymmetric partner

$$
\begin{aligned}
\tilde{R} &= 2 * \mathrm{d}\tilde{\omega} = 2\epsilon^{mn}\partial_n\tilde{\omega}_m\,, && \text{(B.13)}\\
\tilde{\sigma}_\alpha &= *(\tilde{D}\psi)_\alpha = \epsilon^{mn}\big(\partial_n\psi_{m\alpha} + \frac{1}{2}\tilde{\omega}_n(\gamma_*\psi_m)_\alpha\big)\,, && \text{(B.14)}\\
\tilde{\omega}_a &= \epsilon^{mn}\partial_n e_{ma} - i\epsilon^{mn}(\psi_n\gamma_a\psi_m)\,. && \text{(B.15)}
\end{aligned}
$$

As in the bosonic theory Y in (B.11) is a Lagrange multiplier, eliminating ϕ according to

$$
\phi = \epsilon^{mn}(2\partial_n A_m + \psi_n\gamma_*\psi_m) =: \tilde{F}\,. \qquad \text{(B.16)}
$$

[33]The name "Casimir" is justified because, introducing the Schouten-Nijenhuis bracket $\{X^I, X^J\} = P^{IJ}$, a Casimir function fulfills $\{X^I, C\} = P^{IJ}\partial C/\partial X^J = 0$ for all I. On a sidenote, the non-linear Jacobi identity (2.12) is then seen to be a simple consequence of the "ordinary" Jacobi identity for this bracket.

JHEP11(2004)021

Then the action (B.11) reduces to

$$\mathcal{S}_{\rm SO}=\frac{1}{2}\int \mathrm{d}^2x e\Big(\tilde{R}\tilde{F}-\tilde{F}^3+2(\chi\tilde{\sigma})+\frac{1}{4}\chi^2+i\tilde{F}\epsilon^{mn}(\chi\gamma_n\psi_m)+\tilde{F}^2\epsilon^{mn}(\psi_n\gamma_*\psi_m)\Big)\,. \tag{B.17}$$

From the gPSM symmetries and the elimination conditions for X^a and torsion one obtains as SUSY transformations in (B.11) (cf. eqs. (2.40)-(2.43) in [19])

$$\delta\phi = \frac{1}{2}(\chi\gamma_*\varepsilon)\,, \tag{B.18}$$

$$\delta\chi^\alpha = -2i\epsilon^{mn}\Big(\partial_n\phi+\frac{1}{2}(\chi\gamma_*\psi_n)\Big)(\varepsilon\gamma_m)^\alpha-\big(\phi^2-Y\big)(\varepsilon\gamma_*)^\alpha\,, \tag{B.19}$$

$$\delta Y \equiv 0\,, \tag{B.20}$$

$$\delta e_m{}^a = -2i(\varepsilon\gamma^a\psi_m)\,, \tag{B.21}$$

$$\delta\psi_{m\alpha} = -(\tilde{D}\varepsilon)_{m\alpha}-\frac{i}{2}\phi(\gamma_m\varepsilon)_\alpha\,, \tag{B.22}$$

$$\delta A = -\varepsilon\gamma_*\psi\,. \tag{B.23}$$

It is emphasized that the choice of the prepotential (2.15) not only determines the bosonic potential (2.16) but also the transformation law of A in (B.23). Because A stems from a component of the Dreibein this is really the correct transformation law. Also all other transformations are as expected.

Elimination of χ from (B.17) yields

$$\chi_\alpha=-4\tilde{\sigma}_\alpha-2i\tilde{F}\epsilon^{mn}(\gamma_n\psi_m)_\alpha=-4\epsilon^{mn}(\hat{D}_n\psi_m)_\alpha\,, \tag{B.24}$$

where $\hat{D}$ is defined as in (2.8). This allows to eliminate $\tilde{\sigma}_\alpha$ in terms of χ_α and a term proportional to $\tilde{F}$. Insertion into the action (B.17) and changing to the notation[34] of [1] after multiplication with $1/(4\pi^2)$ establishes

$$\mathcal{S}_{\rm SO}=-\frac{1}{8\pi^2}\int \mathrm{d}^2x\sqrt{-g}\Big(r\tilde{F}+\tilde{F}^3+\frac{1}{4}\chi^2-\tilde{F}^2\epsilon^{mn}\psi_n\gamma_*\psi_m\Big)\,. \tag{B.25}$$

The transformations (B.21)–(B.23) are unchanged except for (B.22) which now reads instead

$$\delta\psi_{m\alpha}=-(\tilde{D}\varepsilon)_{m\alpha}-\frac{i}{2}\tilde{F}(\gamma_m\varepsilon)_\alpha=-(\hat{D}_m\epsilon)_\alpha\,. \tag{B.26}$$

The action presented in the main text, eq. (2.17), is a direct consequence of (B.25) together with (B.24).

The action (B.25) can be obtained from a superspace formulation as well. This can be seen from a result of ref. [19] where it was shown that $N=(1,1)$ gPSM models with vanishing bosonic torsion can be mapped onto superspace actions of the model of Howe [23]. In the former case the independent spin connection, X^a and χ^α can be eliminated yielding a theory formulated in terms of dilaton, zweibein and gravitino and completely determined by the prepotential $u(\phi)$. The superspace fomulation consists of one multiplet $(e_m^a,\psi_m^\alpha,\mathcal{A})$,

[34] Note especially $\tilde{R}=-r$, $\tilde{F}=f+\epsilon^{mn}\psi_n\gamma_*\psi_m$, $A=a/2$ and $e=\sqrt{-g}$.

where $\mathcal{A}$ is the auxiliary field. The action is determined by a function $\mathcal{F}(S)$ with S being a scalar superfield that has $\mathcal{A}$ as its lowest component (cf. [23], we adopt the notation of [19]). The two theories are equivalent with the identification

$$\mathcal{A} = -\frac{u'(\phi)}{2}, \qquad \mathcal{F}(\mathcal{A}) = \frac{1}{2}\Big(u\big(\phi(\mathcal{A})\big) - \phi(\mathcal{A})u'\big(\phi(\mathcal{A})\big)\Big). \tag{B.27}$$

Although our model is not equivalent to the one of ref. [19] due to the additional fields Y and A, the identification still turns out to be straightforward. The condition $\mathcal{A} = -\phi$ obviously is globally defined and independent of Y. This does not apply to $\mathcal{F}(\mathcal{A}) = -1/2(\mathcal{A}^2 + Y)$, but it is easily seen e.g. from (B.11) that the Y-dependent terms simply reproduce the constraint (B.16). Thus the action (B.25) is—up to overall factors—equivalent to the superspace action (2.18) if the auxiliary field $\mathcal{A}$ is interpreted as the dual field strength according to (B.16).

C. Conformal transformations

Dilaton dependent (super-)conformal transformations [23, 21]

$$g \to e^{Q(X)} g \tag{C.1}$$

on the world sheet correspond—up to a redefinition of the gravitino—to target space diffeomorphisms at the level of the gPSM [19]. They introduce (or change) the second potential Z in the bosonic part of the action (cf. the eq. in footnote 9), which is related to the conformal factor by means of

$$Z = \frac{\mathrm{d}}{\mathrm{d}X} Q\,. \tag{C.2}$$

With the redefinition

$$\mathrm{d}\tilde{X} = e^{Q}\,\mathrm{d}X \tag{C.3}$$

the transformed line element attains the form

$$\mathrm{d}\tilde{s}^2 = 2\,\mathrm{d}u\,\mathrm{d}\tilde{X} + \underbrace{e^{Q} K(X;C,c)}_{:=\tilde{K}(\tilde{X};C,c)}\,\mathrm{d}u^2\,, \tag{C.4}$$

where X in the Killing norm may be expressed as a function of $\tilde{X}$ by integrating (C.3). It should be noted that the derivative of the Killing norm

$$\frac{\mathrm{d}}{\mathrm{d}\tilde{X}}\tilde{K} = e^{-Q}\frac{\mathrm{d}}{\mathrm{d}X}\left(e^{Q}K\right) = KQ' + \frac{\mathrm{d}}{\mathrm{d}X}K \tag{C.5}$$

is conformally invariant only at Killing horizons $\tilde{K} = 0 = K$. Consequently, Hawking temperature as derived naively from surface gravity is conformally invariant.

For simplicity the focus will be on conformal factors monomial in the dilaton,

$$e^{Q} = \alpha X^{\beta} \quad \to \quad Z = \frac{\beta}{X}\,, \tag{C.6}$$

JHEP11(2004)021

with $\beta \neq -1$. Strictly speaking, one should require positivity, but as the explicit examples below will exclusively be restricted to even β and positive α we can avoid the introduction of absolute values in (C.6). The transformed Killing norm (3.22),

$$\tilde{K} = \alpha \left(\frac{\beta+1}{\alpha}\tilde{X}\right)^{\beta/(\beta+1)} \left[2C + \frac{1}{4}\left(\left(\frac{\beta+1}{\alpha}\tilde{X}\right)^{2/(\beta+1)} - c\right)^2\right], \tag{C.7}$$

leads to the transformed scalar curvature

$$\begin{aligned} \tilde{r} = \alpha\Bigg[& 2\left(C + \frac{c^2}{8}\right)\frac{\beta}{(\beta+1)^2}\left(\frac{\beta+1}{\alpha}\right)^{\beta/(\beta+1)} \tilde{X}^{-(\beta+2)/(\beta+1)} + \\ & + \frac{c}{2}\left(\frac{\beta+1}{\alpha}\right)^{(\beta+2)/(\beta+1)} \frac{\beta+2}{(\beta+1)^2}\tilde{X}^{-\beta/(\beta+1)} - \\ & - \frac{3}{4}\left(\frac{\beta+1}{\alpha}\right)^{(\beta+4)/(\beta+1)} \frac{\beta+4}{(\beta+1)^2}\tilde{X}^{-(\beta-2)/(\beta+1)}\Bigg], \end{aligned} \tag{C.8}$$

which is slightly more complicated than (3.23). The term in the last line provides curvature of the ground state geometry $C = c = 0$. There are now a couple of interesting special cases.

Minkowski ground state. For $\beta = -4$ curvature of the ground state vanishes identically. This can happen only if the ground state geometry is either Minkowski or Rindler space. To achieve the latter the ground state Killing norm has to be linear in $\tilde{X}$, which is not possible. Rather, the Killing norm is constant and thus one obtains a minkowskian ground state. Incidentally, it should be noted that in the original frame the ground state geometry is given by **B1b** (see figure 1). Thus, the singularity of the conformal factor at $X = 0$ is responsible for the absence of horizons in the ground state solution in the Minkowski ground state frame. Minkowski ground state frames are very natural for energy definitions as the notion of "ADM mass" makes sense. As can be deduced from (C.7) the ADM mass, up to a scale factor, will be given by $C + c^2/8$. Thus — exactly like for the extremal RN BH — all BPS kinks have the same BPS mass $C = 0$ but different ADM masses, depending on the charge (cf. footnote 23). So is there anything to be said against the use of this frame? The main objection is not the fact that the conformal factor is singular at $X = 0$, because this could be worked around by allowing binomial (or more complicated) conformal factors which only asymptotically behave monomial, but the fact that the asymptotic region is geodesically incomplete. While the application of this frame is excellent for purposes which involve a "far field approximation" like for Schwarzschild or the RN BH, it becomes less attractive if the asymptotic region turns out to be not asymptotic after all. Thus, unfortunately, for the model under consideration this frame is not very useful.

Dual frame. For $\beta = -2$ the term in the second line of (C.8) vanishes. The name "dual frame" has been chosen because $\tilde{X} \propto 1/X$ (we recall that on-shell $X = f$). Thus, for "strong coupling" (large values of the dual field strength f and thus of the dilaton field X) in the original frame we encounter small values of the transformed dilaton field $\tilde{X}$.

JHEP11(2004)021

Original frame. Trivially, for $\beta = 0$ one remains in the original frame. We would like to pinpoint that only in this frame the contribution in the first line of (C.8) vanishes. This is the "somewhat counter intuitive feature" discussed below eq. (3.23) and thus we see that it is truly an artifact of that frame.

AdS ground state. For $\beta = 2$ ground state geometry has constant curvature r_0. Because α has to be positive $r_0 < 0$ and thus the ground state is AdS. This is a very nice behavior because also the SUSY preserving constant dilaton vacua are AdS and thus in such a frame the kink solution — if it existed — could be matched in the asymptotic region with the constant dilaton vacua *without induced matter fluxes*. Actually, this is very much in the spirit of the original work [1] where no matter fluxes are present because there geometry does not extend over the horizons. As this case is the most relevant for our purposes an explicit expression for the Killing norm will be provided,

$$\tilde{K} = \alpha \left(\frac{3}{\alpha}\tilde{X}\right)^{2/3} \left[2C + \frac{1}{4}\left(\left(\frac{3}{\alpha}\tilde{X}\right)^{2/3} - c\right)^2\right], \tag{C.9}$$

and for the associated scalar curvature

$$\tilde{r} = \alpha \left[\left(C + \frac{c^2}{8}\right)\frac{4}{9}\left(\frac{3}{\alpha}\right)^{2/3}\tilde{X}^{-4/3} + \frac{2c}{9}\left(\frac{3}{\alpha}\right)^{4/3}\tilde{X}^{-2/3}\right] + r_0\,, \tag{C.10}$$

containing the ground state curvature

$$r_0 = -\frac{9}{2\alpha}\,. \tag{C.11}$$

Note that the original asymptotic region $X = \pm\infty$ still corresponds to the transformed asymptotic region $\tilde{X} = \pm\infty$. Thus, any conformal factor which asymptotically behaves monomially as in (C.6) with $\beta = 2$ will exhibit the desirable feature of a ground state geometry which asymptotically is AdS.

Final remark. As can be argued on general grounds a monomial conformal factor (C.6) will always destroy the kink solution (unless $\beta = 0$). This can be checked more explicitly by noticing that for $\beta < -1$ the origin is mapped to infinity and thus the kink may not pass through it. On the other hand, for $\beta > -1$ the exponent of $\tilde{X}$ in the first term in (C.8) is always negative. Thus, a curvature singularity is introduced at the origin (unless $\beta = 0$ where this term vanishes identically) and again the kink solution may not pass through. Therefore, in the main text a suitable conformal factor binomial in X is studied which still retains the favorable features of the last case above.

Note added in proofs. Let us address stability of the kink. To this end we recall that generically the Casimir function (3.18) evaluated at a Killing horizon reads

$$C|_{\text{hor}} = -\frac{1}{8}u^2\bigg|_{\text{hor}} \leq 0\,. \tag{C.12}$$

Thus, if C were positive no horizon could occur and one would encounter naked singularities. Consequently, eliminating solutions exhibiting naked singularities we have shown

JHEP11(2004)021

that mass has to be positive, $M = -C \geq 0$. Because for the kink $C = 0$ it is the physical solution of lowest energy and thus stable.

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JHEP11(2004)021